

BIMALA PRASHAD CHALIHA COLLEGE

PRACTICAL NOTE BOOK
DEPARTMENT OF MATHEMATICS

Project on
Gamma Function



Submitted by

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Class : B.sc 6th semester

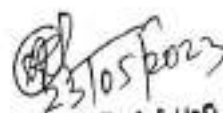
Subject: Mathematics (Honours)

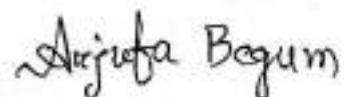
Paper: MAT - HC- 6026

GURoll no: US - 201- 003- 0006

Registration no: 20001731

Year: 2023


Associate Prof. & HoD
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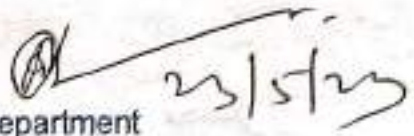
23/05/2023.

CERTIFICATE

This is certified that **Arjuna Begum**, a student of B.Sc 6th semester Mathematics (Honours) has carried out this project report " **GUMMA FUNCTION** by herself. She has fulfilled the requirements for the preparation of the project report for 4th paper of B.sc 6th semester examination.

She worked very hard to prepare this project. I wish him very success in her life.

Signature of Head of the Department


Associate Prof. & HoD
Dept. of Mathematics
B.P. Chaliha College, Nagarbera

Date: 23/5/23

Place: Nagarbera

ACKNOWLEDGEMENT

This Project report entitled " GUMMA FUNCTION" is intended for drawing attention of mathematical application in the field of

My sincere thanks to Dr. Atowar Rahman Head of the department, mathematics of B.P.C College , Nagerbera. for providing nessesary facilities to carry out my project report.

My sincere thanks to Dr. Deepjyoti Borgohain assistant professor of B.P.C College, Nagerbera for this guidance to carry out my project report.

Arjufa Begum.

ARJUFA BEGUM

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Place: Nagerbera

CERTIFICATE

Arjufa Begum a student of B.P.C Chaliha College in the Department of Mathematics (Hon) bearing Roll.No : Us- 201-003-0006 and Registration no: 20001731 of 2020-21 has submitted the the project report entitled " GUMMA FUNCTION" and appeared in viva-voce before the Evolution Committee and presented the project report.

Signature

Evolution Comittee:

- 1.....Chairman / External
- 2.....H.O.D
- 3.....Supervisor

28/5/23
23/5/23
Arjufa Begum
23/05/23

CERTIFICATE OF ORIGINALTY

This certified that **Arjufa Begum**, a student of B.Sc 6th semester Mathematics (Hon) has carried out is report "GUMMA FUNCTION" under my supervision. She has fulfilled all requirements for preparation of the project report for 4th paper of B.Sc 6th semester examination.

She worked very hard prepare this project. I wish him very success in his life.



Signature of supervision

Date: 23/05/23

Place: Nagaribera

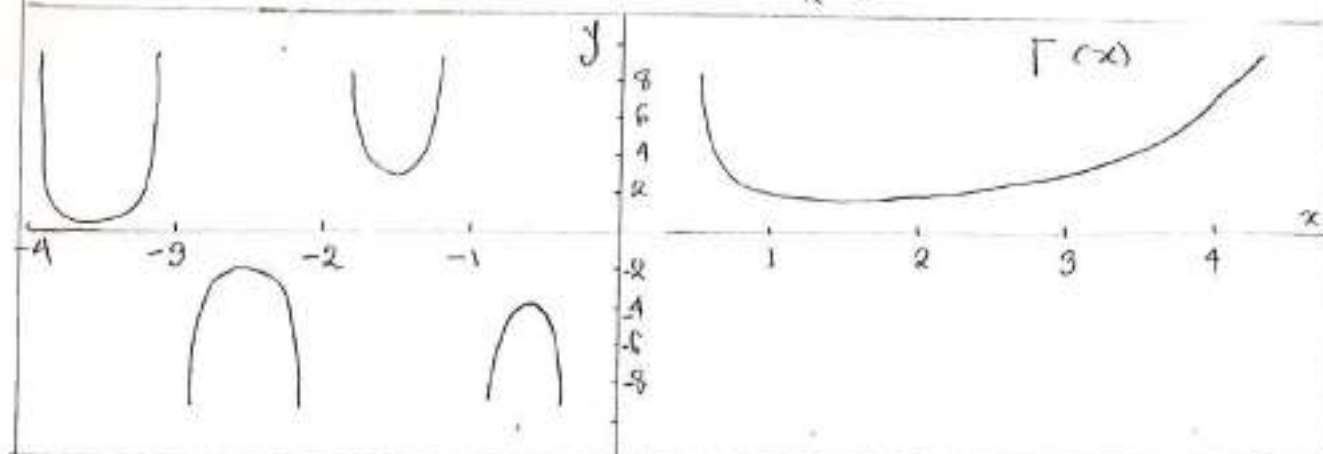
The Gamma function $\Gamma(x)$, is defined as

$$\Gamma(x) \equiv \int_0^{\infty} t^{x-1} e^{-t} dt. \leftarrow$$

where $x \neq \dots, -4, -3, -2, -1, 0$.

Gamma function common rules and facts

- $\Gamma(x+1) \equiv x\Gamma(x) \quad / \quad x \in \mathbb{R}$
- $\Gamma(n+1) \equiv n! \quad n \in \mathbb{N}$ or $\Gamma(n) \equiv (n-1)! \quad n \in \mathbb{N}$
- $\Gamma(1) = 1, \Gamma(2) = 1, \Gamma(\frac{1}{2}) = \sqrt{\pi}$
- $\Gamma(0) = \pm \infty, \Gamma(-1) = \pm \infty, \Gamma(-2) = \pm \infty, \Gamma(-3) = \pm \infty, \text{etc}$
- $\Gamma(z)\Gamma(1-z) \equiv \frac{\pi}{\sin \pi z}, \quad z \in \mathbb{C}$
- $\Gamma'(x) = \Gamma(x) \left[-\gamma + \frac{1}{x} + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{x+k} \right) \right]$



Question-1

Evaluate each of the following expressions, leaving the final answer in exact simplified form.

a) $\frac{3\Gamma(6)}{\Gamma(4)}$

b) $\Gamma\left(\frac{3}{2}\right)$

c) $\int_0^\infty x^7 e^{-x} dx$.

$\forall, 60, \frac{1}{2}\sqrt{\pi}, 7! = 5040$

a) $\frac{3\Gamma(6)}{\Gamma(4)} = \frac{3 \times 5 \Gamma(5)}{\Gamma(4)} = \frac{15 \times 4 \cancel{\Gamma(4)}}{\cancel{\Gamma(4)}} = 60 //$

b) $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{1}{2} \sqrt{\pi} //$

c) $\int_0^\infty x^7 e^{-x} dx \quad \int_0^\infty x^{8-1} e^{-x} dx = \Gamma(8) = 7! //$

Question - 2

Evaluate each of the following expressions, leaving the final answer in exact simplified form.

a) $\frac{60\Gamma(5)}{\Gamma(7)}$.

b) $\Gamma\left(\frac{5}{2}\right)$.

c) $\int_0^{\infty} 2x^4 e^{1-x} dx$.

$V, 2, \frac{3}{4}\sqrt{\pi}, 48e$

a) $\frac{60\Gamma(5)}{\Gamma(7)} = \frac{60\Gamma(5)}{6\Gamma(6)} = \frac{60\cancel{\Gamma(5)}}{6 \times 5\cancel{\Gamma(5)}} = 2 //$

b) $\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2} \times \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{3}{4}\sqrt{\pi} //$

c) $\int_0^{\infty} 2x^4 e^{1-x} dx = \int_0^{\infty} 2x^{5-1} e^1 e^{-x} dx = 2e$

$\int_0^{\infty} x^{5-1} e^{-2} dx = 2e\Gamma(5) = 2e \times 4! = 48e //$

Question 3.

Evaluate each of the following expression, leaving the final answer in exact Simplified form.

a) $\frac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})}$.

b) $\Gamma(-\frac{3}{2})$.

c) $\int_0^{\infty} x^3 e^{-4x} dx$.

$\sqrt{-2}, \frac{4}{3} \sqrt{\pi}, \frac{3}{128}$

a) $\frac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})} = \frac{-\frac{1}{2} \Gamma(-\frac{1}{2})}{-\frac{1}{2} \Gamma(\frac{1}{2})} = \frac{\Gamma(-\frac{1}{2})}{-\frac{1}{2} \Gamma(\frac{1}{2})} = \frac{1}{-\frac{1}{2}} = -2$

OR $\frac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})} = \frac{\cancel{\Gamma(-\frac{1}{2})}}{-\frac{1}{2} \cancel{\Gamma(-\frac{1}{2})}} = \frac{1}{-\frac{1}{2}} = -2 //$

b) $\Gamma(-\frac{3}{2}) \times (-\frac{3}{2}) = \Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = -\frac{3}{2} \Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = -\frac{3}{2} \times (-2) \times (-\frac{1}{2}) \Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = \frac{4}{3} \Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = \frac{4}{3} \sqrt{\pi} //$

$$e) \int_0^{\infty} x^3 e^{-4x} dx = \dots \text{Substitution} \dots = \int_0^{\infty} \left(\frac{1}{4}t\right)^3 e^{-t} \frac{dt}{4}$$

$$= \frac{1}{256} \int_0^{\infty} t^3 e^{-t} dt = \frac{1}{256} \int_0^{\infty} t^{4-1} e^{-t} dt$$

$$= \frac{1}{256} \Gamma(4) = \frac{1}{256} \times 3! = \frac{6}{256}$$

$$= \frac{3}{128}$$

$$t = 4x$$

$$\frac{dt}{dx} = 4$$

$$dx = \frac{dt}{4}$$

Question - 21

By using techniques involving the Gamma function, find the value of

$$\int_0^{\infty} x^3 e^{-\frac{1}{2}x^2} dx.$$

V, IM3, 2

This can be turned into a Gamma function by Substitution

$$\int_0^{\infty} x^3 e^{-\frac{1}{2}x^2} dx = \int_0^{\infty} x^3 e^{-u} \left(\frac{du}{x} \right)$$

$$= \int_0^{\infty} x^2 e^{-u} du = \int_0^{\infty} 2u e^{-u} du$$

$$\Gamma(x) = \int_0^{\infty} x^{x-1} e^{-x} dx$$

$$= 2 \int_0^{\infty} u^{2-1} e^{-u} du = 2 \Gamma(2)$$

$$= 2 \times 1! = 2 //$$

$$\begin{array}{l} u = \frac{1}{2} x^2 \\ du = x dx \\ dx = \frac{du}{x} \\ \hline 2u = x^2 \\ x = \sqrt{2u} \\ \hline \end{array}$$

Question - 5

By using techniques involving the Gamma function, find the value of

$$\int_0^{\infty} \sqrt{x} e^{-\sqrt{x}} dx.$$

V, 4

$$\begin{aligned}\int_0^\infty \sqrt{x} e^{-\sqrt{x}} dx &= \dots \text{By Substitution} \dots \\ &= \int_0^\infty t e^{-t} (2t dt) \\ &= 2 \int_0^\infty t^2 e^{-t} dt = 2 \int_0^\infty t^{3-1} e^{-t} dt \\ &= 2 \Gamma(3) = 2 \times 2! = 4 //\end{aligned}$$

$$\begin{array}{l} t = \sqrt{x} \\ t^2 = x \\ 2t dt = dx \\ \dots \dots \dots \end{array}$$

Question-6

By using techniques involving the Gamma function, find the value of

$$\int_1^\infty \frac{(\ln x)^3}{x^2} dx.$$

V, 6

$$\begin{aligned}
 \int_1^{\infty} \frac{(\ln x)^3}{x^2} dx &= \dots \text{ by Substitution } \therefore = \int_0^{\infty} -\frac{u^3}{x^2} \\
 &\quad (x du) \\
 &= \int_0^{\infty} \frac{u^3}{x} du = \int_0^{\infty} \frac{u^3}{e^u} du = \int_0^{\infty} u^3 e^{-u} du \\
 &= \int_0^{\infty} u^{4-1} e^{-u} du = \Gamma(4) = 3! = 6 //
 \end{aligned}$$

$$\begin{aligned}
 u &= \ln x \\
 du &= \frac{1}{x} dx \\
 dx &= x du \\
 x &= e^u \\
 \dots \dots \dots \\
 x=1 &\mapsto u=0 \\
 x=\infty &\mapsto u=\infty
 \end{aligned}$$

Question-7

By using techniques involving the Gamma function, find the exact value of

$$\int_0^{\infty} 2\sqrt{x} e^{-x^2} dx.$$

Give the answer in the form $\Gamma(k)$, where k is a rational constant.

$$V, \Gamma\left(\frac{3}{4}\right)$$

$$\int_0^\alpha 2\sqrt{x} e^{-x^2} dx = \dots \text{By Substitution} \dots = \int_0^\alpha dt \frac{1}{4} e^{-t} \left(\frac{dt}{2\sqrt{x}} \right)$$

$$= \int_0^\alpha t^{\frac{1}{4}} e^{-t} \frac{dt}{2} = \int_0^\alpha t^{\frac{1}{4}} e^{-t} \frac{dt}{2^{\frac{1}{2}}}$$

$$= \int_0^\alpha t^{\frac{1}{4}} e^{-t} dt = \int_0^\alpha t^{\frac{5}{4}-1} e^{-t} dt$$

$$= \Gamma\left(\frac{5}{4}\right) //$$

$$\begin{array}{l} t = x^2 \\ t^{\frac{1}{2}} = x \\ dt = x \cdot \frac{1}{2} \\ \hline dt = 2x dx \\ \hline \end{array}$$

Question - 8

By using techniques involving the Gamma function, find the exact value of

$$\int_0^\alpha x^6 e^{-4x^2} dx$$

Give the answer in the form $K\sqrt{\pi}$, where K is a rational constant.

$$\sqrt{\frac{15\sqrt{\pi}}{2048}}$$

$$\int_0^{\infty} x^6 e^{-4x^2} dx = \dots \text{By Substitution} \dots = \int_0^{\infty} x^6 e^{-t} \frac{dt}{8x}$$

$$= \frac{1}{8} \int_0^{\infty} x^5 e^{-t} dt = \frac{1}{8} \int_0^{\infty} \left(\frac{t^{1/2}}{2}\right)^5 e^{-t} dt$$

$$= \frac{1}{256} \int_0^{\infty} t^{5/2} e^{-t} dt = \frac{1}{256} \int_0^{\infty} t^{7/2 - 1} e^{-t} dt$$

$$= \frac{1}{256} \Gamma\left(\frac{7}{2}\right) = \frac{1}{256} \times \frac{5}{2} \Gamma\left(\frac{5}{2}\right)$$

$$= \frac{1}{256} \times \frac{5}{2} \times \frac{3}{2} \Gamma\left(\frac{3}{2}\right) = \frac{1}{256} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{15}{2048} \Gamma\left(\frac{1}{2}\right) = \frac{15\sqrt{\pi}}{2048}$$

$$\begin{aligned} t &= 4x^2 \\ dt &= 8x dx \\ dx &= \frac{dt}{8x} \\ x^2 &= \frac{t}{4} \\ x &= \frac{t^{1/2}}{2} \\ \dots \dots \dots \end{aligned}$$

Question - 9

By using techniques involving the Gamma function, find the exact value of

$$\int_0^1 \left[\ln\left(\frac{1}{x}\right) \right]^{a-1} dx$$

Where $a \neq 1, 0, -1, -2, -3, \dots$

Give the answer in terms of a Gamma function.

$$\Gamma(a)$$

• Starting with a Substitution

$$\Rightarrow t = \ln\left(\frac{1}{x}\right)$$

$$\Rightarrow t = -\ln x$$

$$\Rightarrow -t = \ln x$$

$$\Rightarrow x = e^{-t}$$

$$\Rightarrow dx = -e^{-t} dt$$

Limits

$$x=0 \rightarrow t=\infty$$

$$x=1 \rightarrow t=0$$

• The integral transforms to

$$\begin{aligned} \int_0^1 \left[\ln\left(\frac{1}{x}\right) \right]^{a-1} dx &= \int_{\infty}^0 t^{a-1} (-e^{-t}) dt \\ &= \int_0^{\infty} t^{a-1} e^{-t} dt \end{aligned}$$

which is the exact definition
of $\Gamma(a)$ //

Question 10

By using techniques involving the Gamma function.
Show that

$$\int_0^1 [\ln x]^n dx = (-1)^n n! , \quad n \in \mathbb{N}.$$

Proof:

$$\int_0^1 (\ln x)^n dx = \text{By Substitution} = \int_1^0 u^n e^u du$$

$$\text{By another Substitution ...} = \int_1^0 (-t)^n e^{-t} (-dt)$$

$$\begin{array}{l} -t = u \\ -dt = du \\ -\infty \rightarrow \infty \\ 0 \rightarrow 0 \end{array}$$

$$= \int_0^1 (-t)^n e^{-t} dt$$

$$= (-1)^n \int_0^1 t^n e^{-t} dt$$

$$= (-1)^n \Gamma(n+1)$$

$$= (-1)^n n!$$

$$\begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \\ dx = x du \\ dx = e^u du \\ \dots \\ x=0, u=-\infty \\ x=1; u=0 \end{array}$$

Question - 11

By using techniques involving the Gamma function. Show that $\int_0^\infty \frac{e^{-\sigma^2}}{\sigma^k} d\sigma = \frac{\sqrt{\pi}}{2k^{\frac{k}{2}}}, k \neq 0$

$$\int_0^\infty \frac{e^{-\sigma^2}}{\sigma^k} d\sigma = \frac{\sqrt{\pi}}{2k^{\frac{k}{2}}}, k \neq 0$$

proof:-

$$\int_0^a \frac{e^{-\frac{k}{\sigma^2}}}{\sigma^3} d\sigma = \dots \text{By Substitution}$$

• let $u = \frac{k}{\sigma^2}$

$$\sigma^2 = \frac{k}{u}$$

$$\sigma = \sqrt{k} u^{-\frac{1}{2}}$$

$$d\sigma = -\frac{1}{2} \sqrt{k} u^{-\frac{3}{2}} du$$

• limits $\sigma=0 \rightarrow u=a$

$\sigma=a \rightarrow u=0$

$$\dots = \int_a^0 \frac{e^{-u}}{\frac{k^{\frac{3}{2}}}{u^{\frac{3}{2}}}} \left(-\frac{1}{2} \sqrt{k} u^{-\frac{3}{2}} du \right) = \int_0^a \frac{u^{\frac{3}{2}} e^{-u}}{k^{\frac{3}{2}}} \times \frac{\sqrt{k}}{2} u^{-\frac{3}{2}} du$$

$$= \frac{1}{2k^{\frac{3}{2}}} \int_0^a e^{-u} u^{\frac{3}{2}} du = \frac{1}{2k^{\frac{3}{2}}} \Gamma\left(-\frac{5}{2}\right) = \frac{1}{2k^{\frac{3}{2}}} \times \frac{3}{2} \times \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{3}{8k^{\frac{3}{2}}} \Gamma\left(\frac{1}{2}\right) = \frac{3\sqrt{\pi}}{8k^{\frac{3}{2}}} //$$

Question-12

Q) Show clearly that

$$\Gamma(n+1) \equiv n\Gamma(n), n \neq 0, n \in -\mathbb{N}.$$

b) Hence show further that

$$\Gamma(n+1) \equiv n!, \quad n \in \mathbb{N}.$$

Proof:-

$$a) \Gamma(n+1) \equiv \int_0^\infty t^{(n+1)-1} e^{-t} dt = \int_0^\infty t^n e^{-t} dt = \dots$$

integration by parts

$$\left\{ \begin{array}{c|c} t^n & t^{n-1} \\ \hline -e^{-t} & e^{-t} \end{array} \right\}$$

$$\begin{aligned} \dots &= \left[-t^n e^{-t} \right]_0^\infty - \int_0^\infty -n t^{n-1} e^{-t} dt = n \int_0^\infty t^{n-1} e^{-t} dt \\ &= n \Gamma(n) // \end{aligned}$$

$$b) \Gamma(n+1) = n \Gamma(n) = n(n-1) \Gamma(n-1) = n(n-1)(n-2) \Gamma(n-2) = \dots$$

$$= \dots n(n-1)(n-2) \dots 4 \times 3 \times 2 \times 1 \times \Gamma(1) = n! \Gamma(1)$$

$$= n! \int_0^\infty t^{1-1} e^{-t} dt = n! \int_0^\infty e^{-t} dt$$

$$= n! [-e^{-t}]_0^\infty = n! [e^{-t}]_0^\infty = n! [1-0] = n! //$$

Question-13

By using techniques involving the Gamma function, Show that

$$\int_0^{\infty} x^m e^{-ax^n} dx = \frac{\Gamma\left(\frac{m+1}{n}\right)}{na^{\frac{m+1}{n}}}$$

Proof: $\int_0^{\infty} x^m e^{-ax^n} dx = \dots$ By Substitution

$$= \int_0^{\infty} \frac{t^{\frac{m}{n}}}{a^{\frac{m}{n}+1}} \left[\left(\frac{t}{u} \right)^{\frac{1}{u}} \right]^{1-n} e^{-t} dt = \frac{1}{na^{\frac{m+u}{u}}}$$

$$\int_0^{\infty} \frac{t^{\frac{m}{n}}}{a^{\frac{m}{n}+1}} \times \frac{t^{\frac{1}{u}}}{a^{\frac{1-u}{u}}} e^{-t} dt$$

$$= \int_0^{\infty} \frac{t^{\frac{m}{n}}}{a^{\frac{m}{n}+1}} e^{-t} \frac{dt}{na^{\frac{1-u}{u}}} = \int_0^{\infty} \frac{1}{a^{\frac{m}{n}} \times na} t^{\frac{m}{n}} x^{1-n-t} dt$$

$$= \frac{1}{na^{\frac{m}{n}+1}} \int_0^{\infty} \left[\left(\frac{t}{u} \right)^{\frac{1}{u}} \right]^{1-n} e^{-t} dt = \frac{1}{na^{\frac{m+u}{u}}} \int_0^{\infty} \frac{t^{\frac{m}{n}}}{a^{\frac{m}{n}+1}}$$

$$\frac{t^{\frac{m}{n}}}{a^{\frac{m}{n}+1}} \times \frac{t^{\frac{1}{u}}}{a^{\frac{1-u}{u}}} e^{-t} dt$$

$$= \frac{1}{na^{\frac{m+u}{n}} a^{\frac{1-u}{u}}} \int_0^{\infty} t^{\frac{m}{n}} \times t^{\frac{1-u}{u}} e^{-t} dt = \frac{1}{na^{\frac{m+1}{u}}}$$

$$\int_0^{\infty} t^{\frac{m-n+1}{n}} e^{-t} dt$$

$$= \frac{1}{na^{\frac{m+1}{u}}} \int_0^{\infty} t^{\frac{m+1}{n}} e^{-t} dt = \frac{1}{na^{\frac{m+1}{n}}} \Gamma\left(\frac{m+1}{n}\right) //$$

$$\begin{aligned}
 d &= dx \\
 dt &= n x^{n-1} dx \\
 dx &= \frac{dt}{n x^{n-1}} \\
 \dots \dots \dots \\
 x=0 &\rightarrow t=0 \\
 x=a &\rightarrow t=a^n \\
 \dots \dots \dots \\
 x^u &= \frac{t}{a} \\
 x &= \left(\frac{t}{a}\right)^{\frac{1}{u}} \\
 x^m &= \left(\frac{t}{a}\right)^{\frac{m}{u}}
 \end{aligned}$$

Question - 14

By using techniques involving the Gamma function, Show that

$$\int_a^k \frac{e^{ax}}{(k-x)^n} dx = e^{ak} a^{n-1} \Gamma(1-n), \quad 0 < n < 1.$$

proof:- $\int_a^k \frac{e^{ax}}{(k-x)^n} dx = \dots$ By Substitution $\dots$

$$= \int_a^0 \frac{e^{a(k-u)}}{u^n} (-du)$$

$$= \int_0^a \frac{e^{ak-u}}{u^n} du = \int_0^a u^{-n} e^{-au} du$$

$= \dots$ Another Substitution

$$= e^{ak} \int_0^a \left(\frac{t}{a}\right)^{-n} e^{-t} \frac{dt}{a} = e^{ak} \int_0^a \frac{t^{-n} e^{-t}}{a^{1-n} \times a} dt$$

$$= e^{ak} \times \frac{1}{a^{1-n}} \int_0^a t^{-n} e^{-t} dt = e^{ak} a^{n-1} \Gamma(1-n) //$$

$$u = k - x$$

$$du = -dx$$

$$-----$$

$$x = -a, u = a$$

$$x = k, u = 0$$

$$x = k - u$$

$$dx = -du$$

$$dx = -du$$

Question - 15

The Gamma function is defined as

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt, x \neq 0, -1, -2, -3, \dots$$

i) Show that for $x > 0$

$$\Gamma(x+1) = x \Gamma(x)$$

ii) Express the integral

$$J(s) = \int_0^{\infty} e^{-ps} p^s dp, s > 0,$$

in terms of Gamma function.

iii) Deduce that

$$\lim_{s \rightarrow \infty} [J(s)] = 1$$

$$\Gamma(s) = \frac{1}{s} \Gamma\left(\frac{1}{s}\right)$$

1) By definition $\Gamma(n) = \int_0^\infty e^{-t} t^{n-1} dt$

This $\Gamma(n+1) = \int_0^\infty e^{-t} t^{(n+1)-1} dt = \int_0^\infty e^{-t} t^n dt$

integrating By parts w.r.t t

$$\begin{array}{c|c} t^n & t^{n-1} \\ \hline -e^{-t} & e^{-t} \end{array}$$

$$\Gamma(n+1) = \left[-t^n e^{-t} \right]_0^\infty + n \int_0^\infty e^{-t} t^{n-1} dt$$

$$\Gamma(n+1) = n \int_0^\infty e^{-t} t^{n-1} dt$$

$$\Gamma(n+1) = n \Gamma(n) //$$

2) $\Gamma(s) = \int_0^\infty e^{-pt} dt$

$$\Gamma(s) = \int_0^\infty e^{-t} \left(\frac{1}{s} p^{1-s}\right) dt$$

$$\Gamma(s) = \int_0^\infty \frac{1}{s} e^{-t} \left(t^{\frac{1}{s}}\right)^{1-s} dt$$

$$\Gamma(s) = \int_0^\infty \frac{1}{s} e^{-t} t^{\frac{1}{s}-1} dt$$

$$\Gamma(s) = \frac{1}{s} \Gamma\left(\frac{1}{s}\right) //$$

$$\text{III) } \lim_{s \rightarrow 1} [\Gamma(s)] = \lim_{s \rightarrow 1} \left[\frac{1}{s} \Gamma\left(\frac{1}{s}\right) \right] = \lim_{s \rightarrow 1} \left[\Gamma\left(\frac{1}{s} + 1\right) \right]$$

$$= \Gamma(1) = 0! = 1 //$$

Question 16

a) Write down the definition of the Gamma function $\Gamma(x)$, for $\text{Re}(x) > 0$.

b) Use the standard recurrence relation of the Gamma function to extend $\Gamma(x)$,

for $\text{Re}(x) \leq 0$, and hence find the residues at the simple poles at $x = -n$, $n = 0, 1, 2, 3, \dots$

$$\frac{(x)^n}{n!}$$

$$a) \Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$$

$$b) \text{ Using the recurrence } \frac{\Gamma(n+1) = x \Gamma(x)}{\Gamma(x) = \frac{\Gamma(n+1)}{x}}$$

Let $x = u - n$, where $n \in \mathbb{N}$ & $0 < u < 1$

$$\Gamma(u-n) = \frac{\Gamma(u-n+1)}{u-n}$$

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{\Gamma(u-n+2)}{u-n+1}$$

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{\Gamma(u-n+2)}{u-n+2}$$

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{u-n+1} \times \frac{\Gamma(u-n+3)}{u-n+2} \dots$$

~~$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{u-n+1} \times \frac{\Gamma(u-n+2)}{u-n+2} \times \frac{1}{u-2} \times \frac{\Gamma(u)}{u-1}$$~~

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{u-n+1} \times \frac{1}{u-n+2} \times \dots \times$$

$$\frac{1}{u-2} \times \frac{1}{u-1} \times \frac{\Gamma(u+1)}{u}$$

$$\Gamma(u-n) = (-1)^n \frac{1}{(n-u)(n-u-1)(n-u-2) \dots (2-u)(1-u)} \times \frac{\Gamma(u+1)}{u}$$

Hence residue

$$\lim_{u \rightarrow 0} [u \Gamma(u-n)]$$

$$= \lim_{u \rightarrow 0} \left[\frac{u(-1)^n \Gamma(u+1)}{(n-u)(u-u)(n-u-2) \dots (2-u)(1-u)^2} \right]$$

$$= \frac{(-1)^n \Gamma(1)}{n \cdot (n-1)(n-2) \dots \times 3 \times 2 \times 1}$$

$$= \frac{(-1)^n}{n!}$$

Question 17

$$I = 2 \int_0^\infty e^{-x^2} dx \text{ and } I = 2 \int_0^\infty e^{-y^2} dy.$$

By finding an Expression for I , show that

$$I\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

Proof:-

$$\begin{aligned} I\left(\frac{1}{2}\right) &= \int_0^\infty t^{\frac{1}{2}-1} e^{-t} dt = \int_0^\infty t^{-\frac{1}{2}} e^{-t} dt = \dots \text{Substitution} \\ &= \dots \int_0^\infty u^{-\frac{1}{2}} e^{-u} (2u du) = 2 \int_0^\infty e^{-u^2} du \end{aligned}$$

$$\text{Now let } I = 2 \int_0^\infty e^{-x^2} dx \text{ and } I = 2 \int_0^\infty e^{-y^2} dy$$

$$I^2 = \left[2 \int_0^\infty e^{-x^2} dx \right] \left[2 \int_0^\infty e^{-y^2} dy \right]$$

$$I^2 = 4 \int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} dx dy$$

$$I^2 = 4 \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$$

$$\begin{aligned} t &= u^2 \\ dt &= 2u du \end{aligned}$$

Now Switch into polar co. ordinates

$$I^2 = 4 \int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^\infty e^{-r^2} r dr d\theta$$

$$I^2 = 4 \int_{\theta=0}^{\frac{\pi}{2}} \left[-\frac{1}{2} e^{-r^2} \right]_0^\infty d\theta$$

$$I^2 = 2 \int_{\theta=0}^{\frac{\pi}{2}} \left[e^{-r^2} \right]_0^\infty d\theta$$

$$I^2 = 2 \int_{\theta=0}^{\pi/2} 1 \, d\theta$$

$$I^2 = 2 \times [\theta]_0^{\pi/2}$$

$$I^2 = \pi$$

$$\therefore I = \sqrt{\pi}$$

$$\Gamma(1/2) = \sqrt{\pi}$$

$$\bullet \quad x^2 + y^2 = r^2$$

$$\bullet \quad dx \, dy = r \, dr \, d\theta$$



$$\begin{aligned} 0 &\leq x < \infty \\ 0 &\leq y < \infty \end{aligned} \quad \left\{ \begin{array}{l} \text{Because} \end{array} \right.$$

$$\begin{aligned} 0 &\leq r < \infty \\ 0 &\leq \theta < \frac{\pi}{2} \end{aligned}$$

Question - 18

It is given that ~~above~~ the following integral converges

$$\int_0^1 [\ln x]^{1/2} \, dx$$

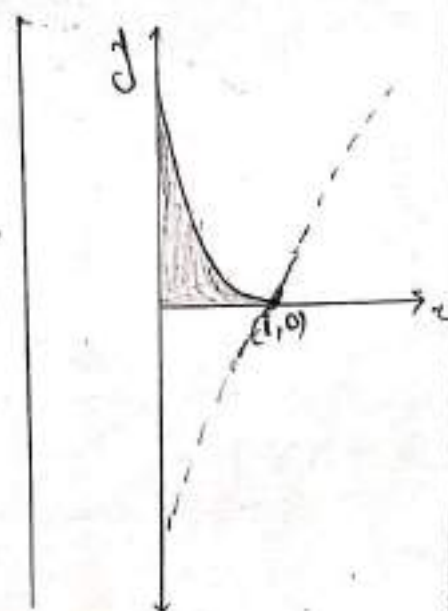
Evaluate the above integral by a Suitable Substitution introducing Gamma functions.

You may assume that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\sqrt{\pi}$$

$$\int_0^1 [\ln x]^{-1/2} \, dx = \dots$$



As $y = \ln x$ is Negative in $[0, 1]$.

$$\int_0^1 [\ln x]^{-1/2} dx = \left[\int_0^1 (\ln x)^{-1/2} dx \right]$$

$$\dots = \left| \int_{\alpha}^0 (\ln x)^{-1/2} dx \right|$$

By Substitution

$$\dots = \left| \int_{\alpha}^0 (-u)^{-1/2} (-e^{-u} du) \right|$$

$$= \left| \int_0^{\pi} (-u)^{-1/2} e^{-u} du \right|$$

$$= \left| \int_0^{\pi} (-1)^{-1/2} e^{-u} du \right|$$

$$= \left| (-1)^{-1/2} \int_0^{\pi} u^{-1/2} e^{-u} du \right|$$

$$= \left| (e^{i\pi})^{-1/2} \int_0^{\pi} e^{-u} u^{-1/2} du \right|$$

$$= \left| e^{-i\pi/2} \int_0^{\pi} e^{-u} u^{-1/2-1} du \right|$$

$$= \left| e^{-i\pi/2} \int_0^{\pi} e^{-u} u^{1/2-1} du \right|$$

$$= \left| e^{-i\pi/2} \right| \left| \Gamma(1/2) \right|$$

$$= \Gamma(1/2) = \sqrt{\pi} //$$

$$\left| \begin{array}{l} \ln x = -u \\ u = \ln x \\ e^{-u} = x \\ dx = -e^{-u} du \\ x=0 \quad u = +\infty \\ x=1 \quad u=0 \end{array} \right|$$

$$\boxed{|e^{i0}| = 1 \forall \theta \in \mathbb{R}}$$

Question - 19

It is given that the following integral Converges

$$\int_0^1 (x \ln x)^n dx, n \in \mathbb{N}.$$

Evaluate the above integral by a Substitution introducing Gamma functions.

$$\dots, \frac{(-1)^n n!}{(n+1)^{n+1}}$$

Start by a Standard Substitution

$u = -\ln x$ ← in order to reverse the log unit to + &

$$\frac{du}{dx} = -\frac{1}{x}$$

$$dx = -x du$$

$$dx = -e^{-u} du \quad \leftarrow \text{Since } -u = \ln x \quad x = e^{-u}$$

for the limits transform

$$x=0 \rightarrow u=\infty$$

$$x=1 \rightarrow u=0$$

Transforming the integral

$$\int_0^1 (x \ln x)^n dx = \int_1^0 [e^{-u} (-u)]^n (-e^{-u} du) = \int_0^1 \frac{(-u)^n}{e^u} e^{-u} du$$

$$= (-1)^n \int_0^1 e^{-nu} e^{-u} u^n du = (-1)^n \int_0^1 e^{-(n+1)u} u^n du$$

linear Simple linear Substitution to Turn into Gamma function

$$t = (n+1)u \iff u = \frac{t}{n+1}$$

$$\implies du = \frac{1}{n+1} dt$$

~~units~~

$$\dots = (-1)^n \int_0^1 e^{-t} \left(\frac{t}{n+1}\right)^n \left(\frac{1}{n+1} dt\right)$$

$$= (-1)^n \times \frac{1}{(n+1)^{n+1}} \int_0^1 e^{-t} t^n dt$$

Using the Definition of the Gamma function

$$\dots = \frac{(-1)^n}{(n+1)^{n+1}} \times \Gamma(n+1)$$

$$= \frac{(-1)^n}{(n+1)^{n+1}} \times n!$$

$$\therefore \int_0^1 (x \ln x)^n dx = \frac{(-1)^n n!}{(n+1)^{n+1}}$$

Question - 20

It is Given that the following integral Converges

$$\int_0^{\infty} e^{-\frac{1}{2}t} \ln t \, dt$$

Evaluate the above integral by introducing a new parametric term in the integrand and carrying out a suitable differentiation under the integral sign.

you may assume that

$$\Gamma'(x) = \Gamma(x) \left[-\gamma + \frac{1}{x} + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{x+k} \right) \right]$$

$$I = \int_0^{\infty} e^{-\frac{1}{2}t} \ln t \, dt = 2(-\gamma + \ln 2)$$

• This has the Signature of a Gamma function

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} \, dt$$

• introduce a parameter x , As in the Gamma function

$$\Rightarrow \text{let } J = \int_0^{\infty} t^{x-1} e^{-\frac{1}{2}t} \ln t \, dt \quad \boxed{\text{Where } I \text{ is } J \text{ with } x=1}$$

• Let observe that $\frac{d}{dx} (t^{x-1}) = t^{x-1} \ln t = t^{x-1} \ln t$

• This can be written as

$$\Rightarrow J = \int_0^{\infty} \frac{J}{J\Gamma} [t^{\Gamma-1} e^{-1/2 t}] dt = \frac{J}{J\Gamma} \left[\int_0^{\infty} t^{\Gamma-1} e^{-1/2 t} dt \right]$$

without is almost a Gamma function

$$\text{Let } u = \frac{1}{2} t \Leftrightarrow t = 2u$$

$$du = \frac{1}{2} dt$$

$$dt = 2 du$$

limits

$$\Rightarrow J = \frac{J}{J\Gamma} \left[\int_0^{\infty} (2u)^{\Gamma-1} e^{-u} (2 du) \right] = \frac{J}{J\Gamma} \left[\int_0^{\infty} 2^{\Gamma} u^{\Gamma-1} e^{-u} du \right]$$

$$\Rightarrow J = \frac{J}{J\Gamma} \left[2^{\Gamma} \int_0^{\infty} e^{-u} u^{\Gamma-1} du \right] = \frac{J}{J\Gamma} \left[2^{\Gamma} \Gamma(\Gamma) \right]$$

• Differentiating the proof

$$\Rightarrow J = 2^{\Gamma} \ln 2 \Gamma(\Gamma) + 2^{\Gamma} \Gamma'(\Gamma)$$

$$\Rightarrow J = J(1) = 2 \ln 2 \Gamma(1) + 2 \Gamma'(1)$$

$$\Rightarrow J = 2 \ln 2 + 2 \Gamma'(1) \quad (\Gamma(1) = 0! = 1)$$

• Now $\Gamma'(\Gamma) = \Gamma(\Gamma) \left[-\gamma - \frac{1}{\Gamma} + \sum_{k=1}^{\infty} \left[\frac{1}{k} - \frac{1}{\Gamma+k} \right] \right]$

$$\Rightarrow \Gamma'(1) = \Gamma(1) \left[-\gamma - 1 + \sum_{k=1}^{\infty} \left[\frac{1}{k} - \frac{1}{k+1} \right] \right]$$

$$\Rightarrow \Gamma'(1) = \gamma - 1 + \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots \right] - \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots \right]$$

$$\Rightarrow \Gamma'(1) = -\gamma - 1 + 1$$

$$\Rightarrow \Gamma'(1) = -\gamma$$

$$\Rightarrow I = 2 \ln 2 + 2(-\gamma)$$

$$\Rightarrow \int_0^{\infty} e^{-\frac{1}{2}t} \ln t \, dt = 2[-\gamma + \ln 2] //$$

Question - 121

prove the validity of Legendre's duplication formula for the Gamma function, which states that

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{\Gamma(2n) \sqrt{\pi}}{2^{2n-1} \Gamma(n)}, \quad n \in \mathbb{N}.$$

proof :-

Start by the ~~revers~~ recursive property of the

Gamma function

$$\Gamma\left(n + \frac{1}{2}\right) = \left(n - \frac{1}{2}\right) \left(n - \frac{3}{2}\right) \left(n - \frac{5}{2}\right) \dots \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} (2n-1) \times \frac{1}{2} (2n-3) \times \frac{1}{2} (2n-5) \dots \left(\frac{1}{2} \times 7\right) \\ \left(\frac{1}{2} \times 5\right) \left(\frac{1}{2} \times 3\right) \left(\frac{1}{2} \times 1\right) \times \sqrt{\pi}$$

$$= \left(\frac{1}{2}\right)^n (2n-1) (2n-3) (2n-5) \dots \cancel{\left(\frac{1}{2}\right)} \cancel{\left(\frac{1}{2} \times 5\right)} \\ \cancel{\left(\frac{1}{2} \times 3\right)} \cancel{\left(\frac{1}{2} \times 1\right)} \times \sqrt{\pi} \\ \dots 7 \times 5 \times 3 \times 1 \times \sqrt{\pi}$$

$$= \frac{1}{2^n} \times \frac{(2n-1)(2n-3)(2n-5) \dots 7 \times 5 \times 3 \times 1}{(2n-2)(2n-4)(2n-6) \dots 6 \times 4 \times 2}$$

$$= \frac{1}{2^n} \times \frac{(2n-1)! \sqrt{\pi}}{2^{n-1} \times (n-1)(n-2)(n-3) \dots \times 3 \times 2 \times 1} \times \sqrt{\pi}$$

$$= \frac{1}{2^n} \times \frac{\Gamma(2n) \sqrt{\pi}}{2^{n-1} (n-1)!}$$

$$= \frac{\Gamma(2n) \sqrt{\pi}}{2^{n-1} (n-1)!} //$$

Question:- 22

Legendre's duplication formula for the Gamma function states that

$$\Gamma(m + 1/2) = \frac{\Gamma(2m) \sqrt{\pi}}{2^{2m-1} \Gamma(m)}, m \in \mathbb{N}.$$

Show that

$$\Gamma(m + 1/2) = \frac{(2m)! \sqrt{\pi}}{2^{2m} m!}, m \in \mathbb{N}.$$

Proof: (By Legendre's duplication formula,

$$\Gamma(m + 1/2) = \frac{\Gamma(2m) \sqrt{\pi}}{2^{2m-1} \Gamma(m)} = \frac{(2m-1)! \sqrt{\pi}}{2^{2m} \times \frac{1}{2} (m-1)!} = \frac{2(2m-1)! \sqrt{\pi}}{2^{2m} (m-1)!}$$

$$= \frac{2m (2m-1)! \sqrt{\pi}}{2^{2m} m (m-1)!} = \frac{(2m)! \sqrt{\pi}}{2^{2m} m!} //$$

Question 23.

$$I(\lambda, x) = \int_0^{\infty} e^{-\lambda t} t^{x-1} \ln t \, dt.$$

(a) By carrying a suitable differentiable over the integral sign, show that.

$$I(\lambda, x) = \lambda^{-x} [F'(x) - \Gamma(x) \ln \lambda].$$

(b) Find simplified expressions for $I(\lambda, 1)$, $I(\lambda, 2)$ and $I(\lambda, 3)$.

$$\text{You may assume that } \Gamma'(x) = \Gamma(x) \left[-\gamma + \frac{1}{x} + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+x} \right) \right].$$

$$I(\lambda, 1) = -\frac{1}{\lambda} [\gamma + \ln \lambda], \quad I(\lambda, 2) = \frac{1}{\lambda^2} [1 - \gamma - \ln \lambda],$$

$$I(\lambda, 3) = \frac{1}{\lambda^3} [3 - 2\gamma - 2 \ln \lambda].$$

a) Start by noting that $d/dx (a^x) = a^x \ln a$.

$$\begin{aligned} \int_0^{\infty} e^{-\lambda t} t^{x-1} \ln t \, dt &= \int_0^{\infty} \frac{\partial}{\partial x} [e^{-\lambda t} t^{x-1}] \, dt \\ &= \frac{\partial}{\partial x} \int_0^{\infty} e^{-\lambda t} t^{x-1} \, dt. \end{aligned}$$

~~As this almost~~

As this almost looks like a Gamma function we use
A simple linear substitution.

$$\begin{aligned} u &= \lambda t \\ \frac{du}{dt} &= \lambda \\ dt &= \frac{1}{\lambda} du \end{aligned}$$

$$= \frac{\partial}{\partial x} \int_0^{\infty} e^{-u} \left(\frac{u}{\lambda} \right)^{x-1} \left(\frac{1}{\lambda} du \right).$$

$$= \frac{\partial}{\partial x} \int_0^{\infty} e^{-u} u^{x-1} \frac{1}{\lambda^x} du.$$

$$= \frac{\partial}{\partial x} \left[\lambda^{-x} \int_0^{\infty} e^{-u} u^{x-1} du \right].$$

$$= \frac{\partial}{\partial x} [\lambda^{-x} \Gamma(x)].$$

Differentiating w.r.t. x , using product rule:

$$\begin{aligned} &= \lambda^{-x} (-1) \ln \lambda \Gamma(x) + \lambda^{-x} \Gamma'(x) \\ &= \lambda^{-x} [\Gamma'(x) - \Gamma(x) \ln \lambda] // \end{aligned}$$

⑥ Start by computing simplified expressed, in terms of γ for the Definition of Gamma for $x = 1, 2, 3, \dots$ using $[\Gamma'(x) = \Gamma(x) [-\gamma - 1/x + \sum_{k=1}^{\infty} (1/k - 1/(x+k))].$

$$\bullet \Gamma'(1) = \Gamma(1) \left[-\gamma - 1 + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right) \right]$$

$$\begin{aligned} &= 0! \left[-\gamma - 1 + 1 - \cancel{1/2} + \cancel{1/2} - \cancel{1/3} + \cancel{1/3} - \cancel{1/4} + \cancel{1/4} - \dots \right] \\ &= 1 [-\gamma - 1 + 1] = -\gamma \end{aligned}$$

$$\bullet \Gamma'(2) = \Gamma(2) \left[-\gamma - 1/2 + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+2} \right) \right]$$

$$= 1! \left[-\gamma - 1/2 + 1 + \cancel{1/2} + \cancel{1/2} + \cancel{1/3} + \cancel{1/4} + \cancel{1/5} + \dots - \cancel{1/3} - \cancel{1/4} - \cancel{1/5} - \dots \right]$$

$$= 1 \cdot [-\gamma - \cancel{1/2} + 1 + \cancel{1/2}] = 1 - \gamma$$

$$\bullet \Gamma'(3) = \Gamma(3) \left[-\gamma - 1/3 + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+3} \right) \right]$$

$$\begin{aligned} &= 2! \left[-\gamma - 1/3 + 1 + \cancel{1/2} + \cancel{1/4} + \cancel{1/5} + \cancel{1/6} + \dots - \cancel{1/4} - \cancel{1/5} - \cancel{1/6} - \dots \right] \end{aligned}$$

$$= 2 \left[-\gamma - \frac{1}{3} + 1 + \frac{1}{2} + \frac{1}{3} \right] = 3 - 2\gamma.$$

finally we obtain:

$$I(\lambda, 1) = \gamma^{-1} [\Gamma'(1) - \Gamma(1) \ln \lambda] = \frac{1}{\gamma} [-\gamma - 0! \ln \lambda] \\ = -\frac{1}{\gamma} (\gamma + \ln \lambda).$$

$$I(\lambda, 2) = \gamma^{-2} [\Gamma'(2) - \Gamma(2) \ln \lambda] = \frac{1}{\gamma^2} [1 - \gamma^2 - 1! \ln \lambda] \\ = \frac{1}{\gamma^2} (1 - \gamma - \ln \lambda)$$

$$I(\lambda, 3) = \gamma^{-3} [\Gamma'(3) - \Gamma(3) \ln \lambda] = \frac{1}{\gamma^3} [(3 - 2\gamma) - 2! \ln \lambda] \\ = \frac{1}{\gamma^3} [3 - 2\gamma - 2 \ln \lambda]$$

Question 24

By considering standard power series expansions and using gamma functions, show

$$\int_0^1 x^r dx = \sum_{r=1}^{\infty} \left[\frac{(-1)^{r-1}}{\gamma^r} \right]$$

Assume that integration and summation commute.

Proof:

Rewriting the integration

$$\int_0^1 x^r dx = \int_0^1 e^{\ln x^r} dx = \int_0^1 e^{r \ln x} dx$$

Rewrite as the exponential series of integrating integration / summation.

$$= \int_0^1 \sum_{r=0}^{\infty} \frac{(x \ln x)^r}{r!} dx = \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^1 (x \ln x)^r dx \right]$$

Now using a substitution (Then minus, to take the limits to 0 to ∞).

$$\begin{aligned} t &= -\ln x \\ -t &= \ln x \\ x &= e^{-t} \\ dx &= -e^{-t} dt \\ x=0 &\mapsto t=\infty \\ x=1 &\mapsto t=0 \end{aligned}$$

$$\begin{aligned} &= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_{\infty}^0 [e^{-t}(-t)]^r (-e^{-t} dt) \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^{\infty} (-x e^{-t})^r e^{-t} dt \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^{\infty} (-1)^r e^{-tr} e^{-t} t^r dt \right] \end{aligned}$$

Now go further. If employ another substitution.

$$\begin{aligned} u &= t(r+1), \quad t = \frac{u}{r+1} \\ du &= dt(r+1) \\ dt &= \frac{du}{r+1} \end{aligned}$$

$$\begin{aligned} &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} t^r e^{-t(r+1)} dt \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} \left(\frac{u}{r+1} \right)^r e^{-u} \left(\frac{du}{r+1} \right) \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} \frac{u^r}{(r+1)^{r+1}} e^{-u} \frac{1}{r+1} du \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \times \frac{1}{(r+1)^{r+1}} \int_0^{\infty} u^r e^{-u} du \right] \end{aligned}$$

Now by definition this is a gamma function

$$\Gamma(x) \equiv \int_0^{\infty} u^{x-1} e^{-u} du \equiv (x-1)!$$

$$\Gamma(x+1) = \int_0^{\infty} u^x e^{-u} du = x!$$

Finally we have,

$$\int_0^1 x^n dx = \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \times \frac{1}{(r+1)^{r+1}} \int_0^{\infty} u^r e^{-u} du \right]$$

$$\int_0^1 x^n dx = \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \times \frac{1}{(r+1)^{r+1}} \times r! \right]$$

$$\int_0^1 x^n dx = \sum_{r=0}^{\infty} \frac{(-1)^r}{(r+1)^{r+1}}$$

$$\therefore \int_0^1 x^n dx = \sum_{r=1}^{\infty} \frac{(-1)^{r-1}}{(r)^{r+1}}$$

$$\int_0^1 x^n dx = \sum_{r=1}^{\infty} \frac{(-1)^{r-1}}{r^s} //$$

$$1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \frac{1}{5^5} - \frac{1}{6^6} + \frac{1}{7^7} - \dots$$

Question 25

By considering standard power series expansions and using Gamma functions, show

$$\int_0^1 \frac{1}{x^n} dx = 1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \frac{1}{5^5} + \dots$$

Assume that integration and summation commute.

Proced As Follows:

Proof:

$$\int_0^1 \frac{1}{x^x} dx = \int_0^1 x^{-x} dx = \int_0^1 e^{\ln x^{-x}} dx = \int_0^1 e^{-x \ln x} dx.$$

Using the expanding series

$$\begin{aligned} \int_0^1 \sum_{r=0}^{\infty} \frac{(-x \ln x)^r}{r!} dx &= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^1 (-1)^r x^r (\ln x)^r dx \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^1 x^r (\ln x)^r dx \right] \end{aligned}$$

Now using a substitution to create a gamma function
(use a minus to get the "correct" limits).

$$\begin{aligned} t &= -\ln x \\ -t &= \ln x \\ x &= e^{-t} \\ dx &= -e^t dt \\ x=0 &\mapsto t=\infty \\ x=1 &\mapsto t=0 \end{aligned}$$

$$\begin{aligned} &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} e^{-tr} (-t)^r (-e^t) dt \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} t^r (-1)^r e^{-tr} e^t dt \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{(-1)^r (-1)^r}{r!} \int_0^{\infty} t^r e^{-t(r+1)} dt \right] \end{aligned}$$

Another substitution to create a "full gamma"

$$\begin{aligned} u &= t(r+1) \\ du &= dt(r+1) \\ t &= \frac{du}{r+1} \\ dt &= \frac{du}{r+1} \end{aligned}$$

$$\begin{aligned} &= \sum_{r=0}^{\infty} \left[\frac{(-1)^{2r}}{r!} \int_0^{\infty} \left(\frac{u}{r+1} \right)^r e^{-u} \left(\frac{du}{r+1} \right) \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^{\infty} \frac{u^r}{(r+1)^{r+1}} e^{-u} \frac{1}{r+1} du \right] \\ &= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \frac{1}{(r+1)^{r+1}} \int_0^{\infty} u^r e^{-u} du \right] \end{aligned}$$

Now this a gamma function.

$$\Gamma(r) \equiv \int_0^{\infty} u^{r-1} e^{-u} du = (r-1)!$$

$$\Gamma(r+1) = \int_0^{\infty} u^r e^{-u} du = r!$$

Thus we have.

$$\int_0^1 \frac{1}{x^x} dx = \sum_{r=0}^{\infty} \left[\frac{1}{r!(r+1)^{r+1}} \Gamma(r+1) \right]$$

$$\int_0^1 \frac{1}{x^x} dx = \sum_{r=0}^{\infty} \left[\frac{1}{r!(r+1)^{r+1}} (r!) \right]$$

$$\int_0^1 \frac{1}{x^x} dx = \sum_{r=0}^{\infty} \frac{1}{(r+1)^{r+1}} = \sum_{r=1}^{\infty} \frac{1}{r^r}$$

$$\therefore \int_0^1 \frac{1}{x^x} dx = 1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \frac{1}{5^5} + \frac{1}{6^6} + \dots //$$

Question 26

$$I = \int_1^{\infty} \frac{(\ln x)^3}{x^2(x-1)} dx$$

Show by the means of partial fractions and suitable substitution that

$$I = -6 + \int_0^{\infty} u^3 e^{-u} (1 - e^{-u})^{-1} du,$$

and hence show that $I = \frac{\pi^4}{15} - 6$

Assume without proof that.

$$\zeta(4) = \sum_{n=1}^{\infty} \left[\frac{1}{n^4} \right] = \frac{\pi^4}{90}$$

$$\int_1^{\infty} \frac{(\ln x)^3}{x^3(x-1)} dx = \frac{\pi^4}{15} - 6$$

Proof:

• Start by partial fraction

$$\frac{1}{x^2(x-1)} \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$$

$$1 \equiv A(x-1) + Bx(x-1) + Cx^2$$

$$1 \equiv (B+C)x^2 + (A-B)x - A$$

$$\therefore A = -1, B = -1, C = 1$$

• Rewrite the Integral

$$\Rightarrow I = \int_1^{\infty} \left[\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \right] (\ln x)^3 dx$$

• By substitution next

$$u = \ln x \quad x=1 \mapsto u=0$$

$$e^u = x \quad x=\infty \mapsto u=\infty$$

$$dx = e^u du$$

$$\Rightarrow I = \int_0^{\infty} \left[\frac{1}{e^u - 1} - \frac{1}{e^u} - \frac{1}{e^{2u}} \right] u^3 (e^u du)$$

$$\Rightarrow I = \int_0^{\infty} \left[\frac{e^u}{e^u - 1} - 1 - \frac{1}{e^u} \right] u^3 du$$

$$\Rightarrow I = \int_0^{\infty} u^3 \left[\frac{e^u - e^{-u} + 1}{e^u - 1} \right] du - \int_0^{\infty} u^3 e^{-u} du$$

$$\Rightarrow I = \int_0^{\infty} u^3 \left(\frac{e^{-u}}{1 - e^{-u}} \right) du$$

$$\Rightarrow I = \int_0^{\infty} \frac{u^3}{e^u - 1} du = \Gamma(4)$$

$$\Rightarrow I = \int_0^{\infty} u^3 \left(\frac{e^{-u}}{1 - e^{-u}} \right) du = 6!$$

$$\Rightarrow I = \int_0^{\infty} u^3 e^{-u} (1 - e^{-u})^{-1} du = 6$$

$$\Rightarrow I = \int_0^{\infty} u^3 e^{-u} [1 + e^{-u} + e^{-2u} + e^{-3u} + \dots] du = 6$$

$$\begin{array}{c} \uparrow \\ \boxed{\begin{array}{l} \frac{1}{1-r} = 1 + r + r^2 + r^3 + \dots \quad |r| < 1 \\ \text{Here, } |e^{-u}| = \left| \frac{1}{e^u} \right| = \frac{1}{e^u} < 1 \end{array}} \end{array}$$

$$\Rightarrow I = \int_0^{\infty} u^3 [e^{-u} + e^{-2u} + e^{-3u} + e^{-4u} + \dots] du = 6$$

$$\Rightarrow I = \int_0^{\infty} u^3 \left(\sum_{r=1}^{\infty} e^{-ru} \right) du = 6$$

$$\Rightarrow I = \sum \int_0^{\infty} u^3 e^{-ru} du = 6$$

• By part 3 times ; Differentiation w.r.t Γ under the integrate sign, or switch into Gamma. By substitution

$$v = \gamma u$$

$$u = \frac{1}{\gamma} v$$

$$du = \frac{1}{\gamma} dv$$

$$\Rightarrow I = \sum_{\gamma=1}^{\infty} \int_0^{\infty} \left(\frac{1}{\gamma} v\right)^3 e^{-v} \frac{1}{\gamma} dv - 6$$

$$\Rightarrow I = \sum_{\gamma=1}^{\infty} \int_0^{\infty} \frac{1}{\gamma^4} v^3 e^{-v} dv - 6$$

$$\Rightarrow I = \sum_{\gamma=1}^{\infty} \frac{1}{\gamma^4} \int_0^{\infty} v^3 e^{-v} dv - 6$$

$$\Rightarrow I = \sum_{\gamma=1}^{\infty} \left[\frac{1}{\gamma^4} \Gamma(4) \right] - 6$$

$$\Rightarrow I = \left[\sum_{\gamma=1}^{\infty} \frac{3!}{\gamma^4} \right] - 6$$

$$\Rightarrow I = 6 \left[\sum_{\gamma=1}^{\infty} \frac{1}{\gamma^4} \right] - 6$$

$$\Rightarrow I = 6 \zeta(4) - 6$$

$$\Rightarrow I = 6 \times \frac{\pi^4}{90} - 6$$

$$\Rightarrow I = \frac{\pi^6}{15} - 6 //$$

$\zeta(4) = \frac{\pi^4}{90}$ can easily be derived by considering the Fourier series expansion of x^2 on $(-\pi, \pi)$ of Parseval identity.

Question - 27

By using the substitution $u = x + u\sqrt{x}$ in the integral definition of $\Gamma(x+1)$, show that for large values of n

$$n! \approx \sqrt{2\pi} e^{-n} n^{n+1/2}, \quad n \in \mathbb{N}.$$

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \quad \& \quad \Gamma(x+1) = \int_0^\infty t^x e^{-t} dt.$$

Proof:

$$\bullet \text{ let, } t = x + u\sqrt{x}$$

$$dt = \sqrt{x} du$$

$$t=0, u = -\sqrt{x}$$

$$t=x, u = 0.$$

$$\Rightarrow \Gamma(x+1) = \int_{-\sqrt{x}}^0 (x + u\sqrt{x})^x e^{-(x+u\sqrt{x})} \sqrt{x} du$$

$$\Rightarrow \Gamma(x+1) = \int_{-\sqrt{x}}^0 x^x \left[1 + \frac{\sqrt{x}}{x} u\right]^x e^{-x} e^{-u\sqrt{x}} n^{1/2} du$$

$$\Rightarrow \Gamma(x+1) = x^{x+1/2} e^{-x} \int_{-\sqrt{x}}^0 e^{-u\sqrt{x}} \left[1 + \frac{u}{\sqrt{x}}\right]^x du$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^0 e^{-u\sqrt{x}} \times e^{\ln\left[1 + \frac{u}{\sqrt{x}}\right]^x} du.$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^0 e^{-u\sqrt{x}} \times e^{x \ln(1 + u/\sqrt{x})} du.$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^0 \exp\left[-u\sqrt{x} + x \ln\left(1 + \frac{u}{\sqrt{x}}\right)\right] du$$

- Now $\ln(1+w) = w - \frac{w^2}{2} + \frac{w^3}{3} - \frac{w^4}{4} + \dots$ $-1 < w \leq 1$

So if $-\sqrt{x} < u \leq \sqrt{x}$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} \int_{-\sqrt{x}}^{\sqrt{x}} \exp \left[-u\sqrt{x} + x \left[\frac{u}{\sqrt{x}} - \frac{u^2}{2x} + \frac{u^3}{3x^{3/2}} - \frac{u^4}{4x^2} + \dots \right] \right] du$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^{\sqrt{x}} \exp \left[-u\sqrt{x} + x \left[\frac{u}{\sqrt{x}} - \frac{u^2}{2x} + \frac{u^3}{3x^{3/2}} - \frac{u^4}{4x^2} + \dots \right] \right] du$$

+

$$\int_{\sqrt{x}}^{\infty} \exp \left[-u\sqrt{x} + x \left[\frac{u}{\sqrt{x}} - \frac{u^2}{2x} + \frac{u^3}{3x^{3/2}} - \frac{u^4}{4x^2} + \dots \right] \right] du$$

- As $x \rightarrow \infty$ the second integrate tends to zero, const. the top units is ∞ .

- As $x \rightarrow \infty$ & Integrate in the first ~~set~~ integral between $\exp(-u^2)$.

$$\text{Thus } \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} \approx \int_{-\infty}^{\infty} e^{-u^2} du \quad \text{As } x \rightarrow \infty$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} \approx \sqrt{2\pi} \quad \text{As } x \rightarrow \infty$$

$$\Rightarrow \Gamma(x+1) \approx \sqrt{2\pi} x^{x+1/2} e^{-x}$$

As $x \rightarrow \infty$.

• let $x = n = \text{positive integer}$.

$$\Gamma(x+1) \approx \sqrt{2\pi} n^{n+1/2} e^{-n}$$

$$n! \approx \sqrt{2\pi} e^{-n} n^{n+1/2} //$$

A PROJECT REPORT
ON

"Study Of Center Of Gravity"

PAPER CODE:- MAT-IIE-6086

IN PARTIAL REQUIREMENT OF FULFILMENT
OF THE DEGREE OF BACHELOR OF SCIENCE
IN MATHEMATICS, 2023



Submitted by:-

Asraful Jakirin

G.U.Roll No:- US-201-003-0007

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Department Of Mathematics

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23/05/2023
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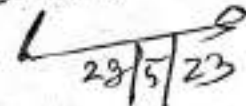
CERTIFICATE

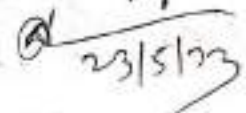
Asraful Jakirin, a student of B.SC 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) bearing Roll No: US-201-003-0007, Reg. No: 20001732 has submitted the project report entitled "**CENTER OF GRAVITY**" and appeared in viva-voce before the evaluation committee and presented the project report.

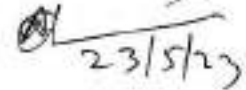
Evaluation Committee

1. Chairman / *External*
2. H.O.D.
3. Supervisor

Signature


23/5/23


23/5/23


23/5/23

CERTIFICATE

This is certified that **Asraful Jakirin**, a student of B.SC 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) has carried out this project report entitled "**CENTER OF GRAVITY**" by himself. He has fulfilled all requirements for the preparation of project report for the 4th paper of B.Sc. (CBCS) 6th semester examination, 2023

He worked very hard to prepare this project report. I wish his very success in his life.

Date: 23/05/2023

Place: Nagarbera

Head of the Department

Associate Prof. & HoD
Deptt. of Mathematics
B.P. Chaliha College, Nagarbera

CERTIFICATE

This is certified that **Asraful Jakirin**, a student of B.SC 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) has carried out. This project report entitled "**CENTER OF GRAVITY**" by himself under my supervision. He has fulfilled all the requirements for the preparation of project report for the 4th paper of B.Sc. (CBCS) 6th semester examination, 2023

He worked very hard to prepare this project
report.

I wish him very success in his life.

Date:.....

23/5/23

Place:.....

~~Asraful Jakirin~~
Nayalun

Supervisor

23/5/23

ACKNOWLEDGEMENT

This project report entitled "**CENTER OF GRAVITY**" is intended for drawing attention of mathematical applications in the field of Partial Differential Equations.

My sincere thanks to professor **Dr. Atowar Rahman**, Head of the Department of mathematics of B.P. Chaliha College, Nagarbera for his guidance & providing necessary facilities to carry out my project report.

My sincere thanks to **Dr. Deepjyoti Borgohain**, Assistant Professor B.P. Chaliha College, Nagarbera for his help to carry out my project report successfully.

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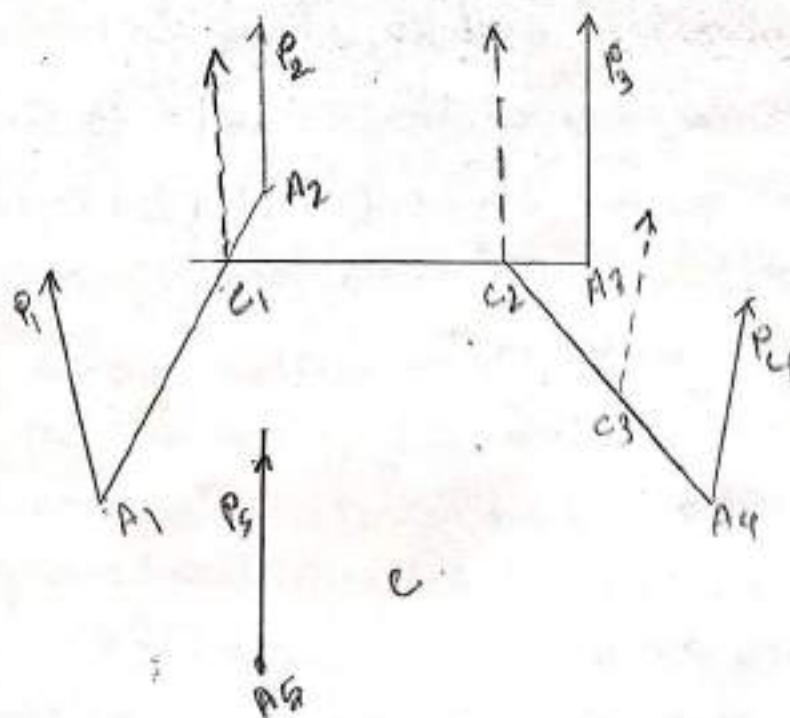
Date: *23/05/2023*

Place: *Nagarbera*

CENTRE OF GRAVITY

1. Centre of like parallel forces -

Let $P_1, P_2, P_3, \dots$ be a set of like parallel forces acting on a rigid body at $A_1, A_2, A_3, \dots$ respectively. Join $A_1 A_2$ and divide it internally at C_1 such that $A_1 C_1 : A_1 C_2 = P_2 : P_1$. Then the resultant of P_1 and P_2 is a parallel force $(P_1 + P_2)$ acting at C_1 . Join $C_1 A_3$ and divide it internally at C_2 such that $C_1 C_2 : C_2 A_3 = P_3 : (P_1 + P_2)$. Then the parallel force $(P_1 + P_2 + P_3)$ acting at C_2 is the resultant of $(P_1 + P_2)$ at C_1 and P_3 at A_3 , that is, of P_1 at A_1 , P_2 at A_2 and P_3 at A_3 . Next join $C_2 A_4$ and divide it at C_3 such that

$$C_2 C_3 : A_3 A_4 = P_4 : (P_1 + P_2 + P_3)$$


Proceeding in this manner till all the forces are exhausted we finally arrive at a point C through which passes a force of magnitude ΣP and parallel to the system, which is the resultant of all the given parallel forces.

Now, it is evident that the positions of C, C_2, C_3 , etc. depend only on the magnitudes of P_1, P_2, P_3 and on the positions of the point A_1, A_2, A_3 where they act, but have nothing to do with the common direction of the parallel forces.

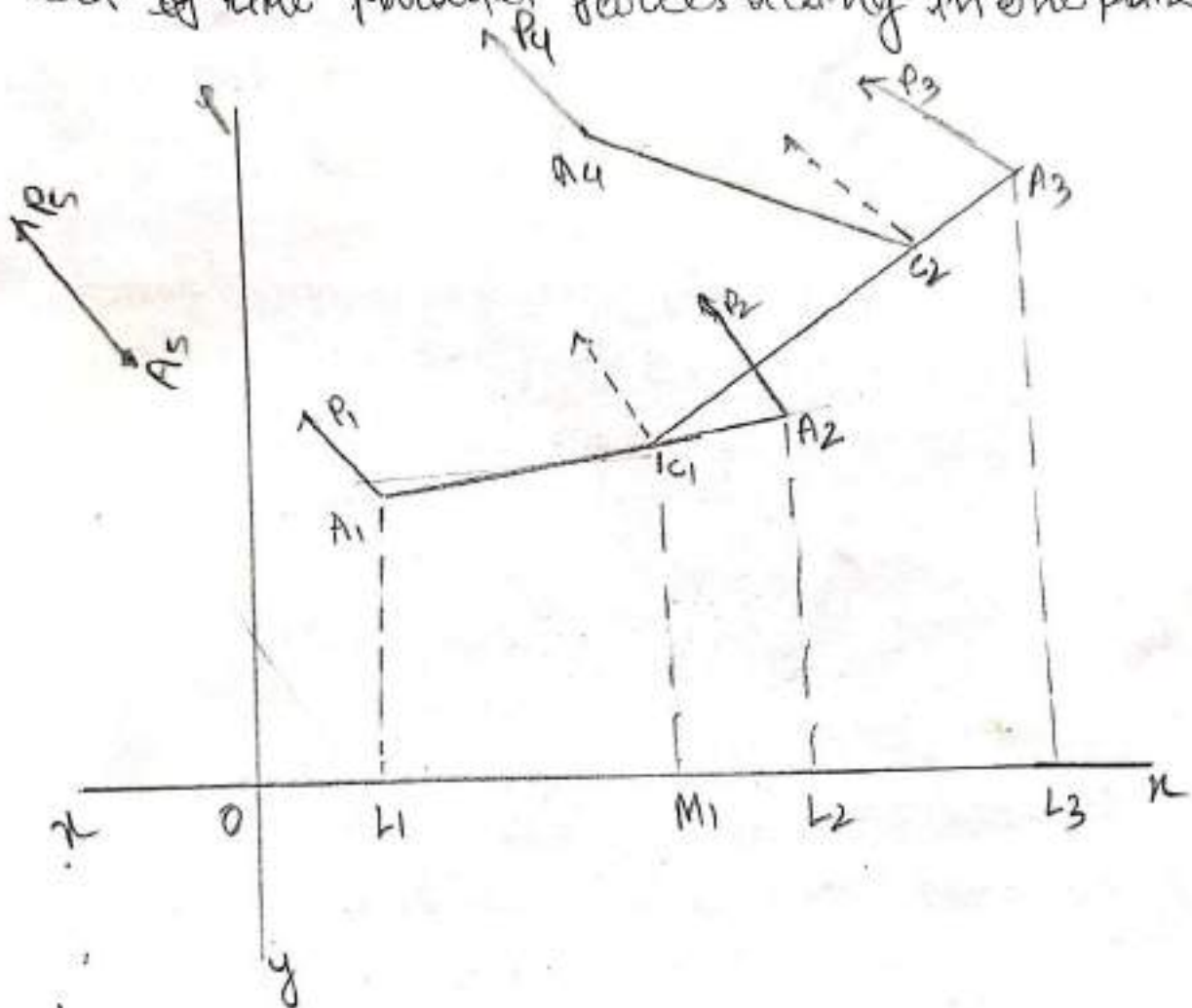
Hence the point C arrived at, through which the final resultant of the parallel forces passes, is fixed, whatever be the common direction of the parallel forces, so long as their magnitudes and points of application remain unchanged. This fixed point C is defined as the centre of the given system of like parallel forces.

We may note that C being the point through which the resultant (magnitude ΣP) always passes, whatever be the common direction of the given parallel forces P_1, P_2, P_3 , - it is a unique point, and will come out to be the same with whatever order we proceed to combine.

The given forces in succession.

Analytical determination.

We shall confine ourselves to the case of a set of like parallel forces acting in one plane.



Let $P_1, P_2, P_3, \dots$ be a set of like parallel forces acting at $A_1, A_2, A_3, \dots$ on a plane, and let $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots$ be the co-ordinates of $A_1, A_2, A_3, \dots$ referred to a set of rectangular axes on the plane.

The resultant of P_1 at A_1 and P_2 at A_2 is $(P_1 + P_2)$ acting at C_1 on A_1A_2 such that $A_1C_1 : C_1A_2 = P_2 : P_1$.

Let ξ_1, η_1 be the co-ordinates of C_1 . Then $A_1 L_1$, $A_2 L_2$ and $C_1 M_1$ being perpendiculars upon OX , since these are parallel lines,

$$\frac{L_2 M_1}{M_1 L_2} = \frac{A_1 C_1}{C_1 A_2}, \text{ that is } \frac{\xi_1 - \kappa_1}{\kappa_2 - \xi_2} = \frac{P_2}{P_1},$$

$$\text{whence we get } \xi_1 = \frac{P_1 \kappa_1 + P_2 \kappa_2}{P_1 + P_2}$$

Exactly in a similar manner, the κ -coordinate (ξ_2 , say) of C_2 where the resultant $(P_1 + P_2 + P_3)$ of $(P_1 + P_2)$ at C_2 and P_3 at A_3 act is given by

$$\xi_2 = \frac{(P_1 + P_2) \xi_1 + P_3 \kappa_3}{(P_1 + P_2) + P_3} = \frac{P_1 \kappa_1 + P_2 \kappa_2 + P_3 \kappa_3}{P_1 + P_2 + P_3} \quad \text{--- (1)}$$

Proceeding in this manner, when all the forces are exhausted the centre of the parallel forces through which the final resultant passes having co-ordinates ξ, η , we get

$$\xi = \frac{P_1 \kappa_1 + P_2 \kappa_2 + P_3 \kappa_3 + \dots}{P_1 + P_2 + P_3 + \dots} = \frac{\sum P \kappa}{\sum P},$$

and similarly,

$$\begin{aligned} \eta &= \frac{P_1 y_1 + P_2 y_2 + P_3 y_3 + \dots}{P_1 + P_2 + P_3 + \dots} \\ &= \frac{\sum P y}{\sum P}. \end{aligned}$$

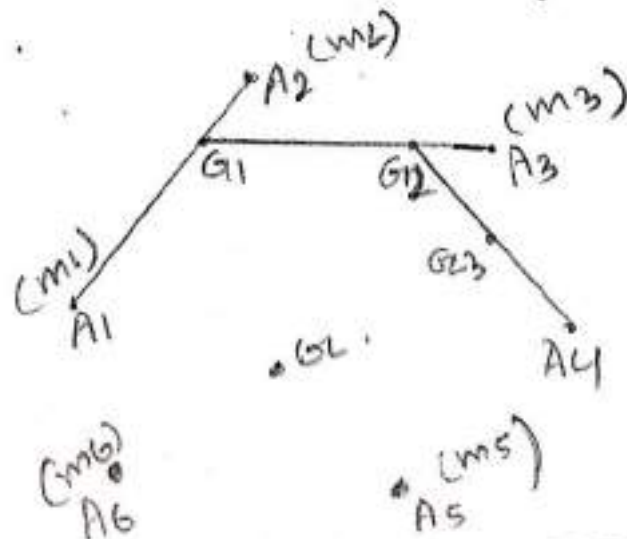
Centre of mass.

Let a system of particles of masses $m_1, m_2, m_3, \dots$ be placed at the point $A_1, A_2, A_3, \dots$

Divide $\overline{A_1 A_2}$ at G_1 in the inverse ratio of the masses at its extremities, that is $A_1 G_1 : G_1 A_2 = m_2 : m_1$. Assume the total mass

$(m_1 + m_2)$ be collected at G_1 . Divide $\overline{G_1 A_3}$ at G_2 in the inverse ratio of the masses at the extremities such that $G_1 G_2 : G_2 A_3 = m_3 : (m_1 + m_2)$.

$G_1, G_2 : G_2 A_3 = m_3 : (m_1 + m_2)$, and assume the total mass at the extremities, that is $(m_1 + m_2 + m_3)$ to be concentrated at G_2 . Proceeding in this manner, when all the particles have been exhausted, we arrive at a final point G which is defined as the centre of mass, or centre of inertia of the given system of particles.



The symmetry of the forms of the co-ordinates also shows that the point G is unique, and will be arrived at finally, in whatever order the masses may be combined in succession.

At instead of discrete particles, we have a finite body of any. may consider it as an agglomeration of an infinite number of infinitely small particles, and define its centre of mass as above. It will be a definite point in the body.

Centre of mean-position.

More generally, let there be a set of points $A_1, A_2, A_3, \dots$ and let $m_1, m_2, m_3, \dots$ be any set of numbers which are associated with these points respectively.

Now join any two points A_1, A_2 and divide $\overline{A_1 A_2}$ at G_1 in the inverse ratio of the numbers associated at the ends such that $A_1 G_1 : G_1 A_2 = m_2 : m_1$.

Assume the number $(m_1 + m_2)$ to be associated with G_1 . Join $G_1 A_3$ and divide it at G_2 such that

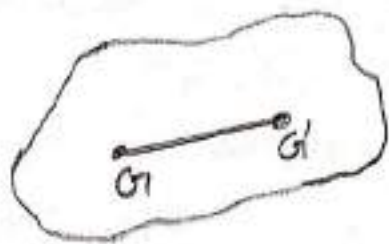
$$G_1 G_2 : G_2 A_3 = m_3 : (m_1 + m_2),$$

and suppose G_2 to be associated with $(m_1 + m_2 + m_3)$. Proceeding in this manner, we find finally arrive at a unique point G which is called the centre of mean-position of the given points for the system of given multipliers.

If the points $A_1, A_2 \dots$ etc. are coplanar with co-ordinates $(x_1, y_1), (x_2, y_2), \dots$ etc, we get exactly as in the previous article, the co-ordinates of G given by

$$\bar{x} = \frac{\sum mx}{\sum m}, \quad \bar{y} = \frac{\sum my}{\sum m}$$

The centre of gravity of a body is unique.



For, if possible, let a body have two centres of gravity, G and G' . The weight of the body passes through both G and G' ; by definition, in whatever manner the body may be held. Now hold the body such that $\overline{GG'}$ is horizontal, the weight, which is a vertical force, cannot now pass through both G and G' unless they coincide. Hence, there cannot be two distinct centres of gravity of the body.

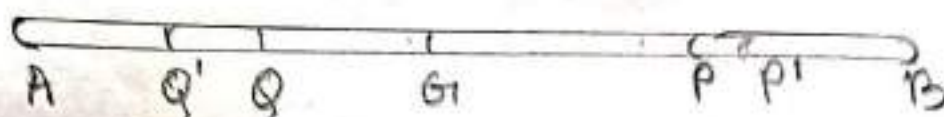
Centre of gravity.

We know from the law of gravitation that every material particle is attracted towards the centre of the earth with a force which is proportional to the mass of the particle, and this force we call its weight.

Now, given any material body, we can consider it to be an assemblage of particles each of which is acted on by the earth's attraction, and all these forces passing through the centre of the earth they have got a single resultant which we call the weight of the body. If the body be held in different positions the positions of the lines of action of this weight or relative to the body will be different. Now, in some cases, the line of action of the weight always passes through a fixed point in the body as, however the body may be placed; for instance, in case of a spherical body by

Determination of centre of gravity in special cases.

I. A thin uniform rod.



Let AB be a thin uniform rod of any material, G the middle point of AB .

Consider two equal infinitesimal lengths PP' and QQ' of the rod equidistant from G , so that $GP = GQ$. Since the rod is

uniform, the weights of these two elements

(which can ultimately be treated as two particles equidistant from G) are equal

both vertically downwards, and the resultant of these two equal and like parallel forces acts at the middle point G .

Since $AG = GB$, the whole rod can be

divided into pairs of such equal infinitesimal elements equidistant from G , and for each pair the resultants of the weights act at G . Hence, the weight of the whole rod acts at the middle point G .

Weight of the rod $\overline{BC}$ acting at its $\overline{AB}$ and $\overline{AC}$ acting at x will act at some point on $\overline{DX}$. Hence, the combined C.G. of the three rods is situated on the line $\overline{DX}$.

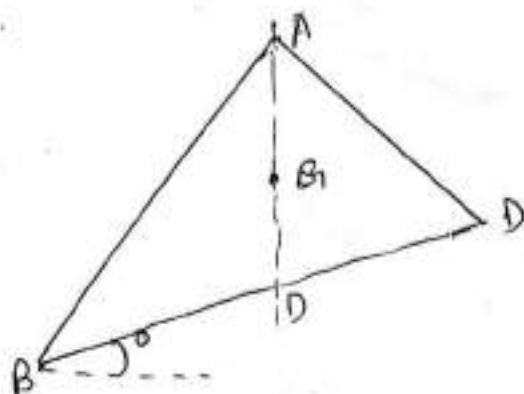
Similarly, combining the weights $\overline{AB}$, $\overline{BC}$ first and then considering the weight of $\overline{AC}$, the combined C.G. of the three rods is shown to be some point on $\overline{EY}$ which bisects the angle $\overline{DEF}$.

Thus, the combined C.G. of the system of three rods, being a common point situated on both $\overline{DX}$ and $\overline{EY}$, is their point of

intersection, that is, the required C.G. is the in-centre of the triangle,

$\overline{DEF}$ formed by joining the middle points of the rods.

Ex. - Triangular lamina ABC hangs at rest, one of the angles A being supported at a fixed point. Find the angle which the lower side makes with the horizon.



D being the mid-point of the lower side BC, the centre of gravity G of the triangle lies on AD. Now as the lamina hangs in equilibrium under the weight acting vertically downwards through G and the reaction at the point of support A, these two forces must be equal and opposite, acting in the same straight line. Thus AGD must be vertical. If θ be the required inclination BC to the horizon, then.

$$\angle ADC = 90^\circ - \theta$$

$$\text{Now, } BD = DC, \therefore \text{Therefore } \frac{BD}{AD} = \frac{DC}{AD}$$

$$\text{or, } \frac{\sin BAD}{\sin ABD} = \frac{\sin CAD}{\sin ACD}$$

$$\text{that is, } \frac{\sin(90^\circ - \theta - B)}{\sin B} = \frac{\sin(\theta - C)}{\sin C}$$

$$\text{or, } \frac{\cos(\theta + B)}{\sin B} = \frac{\cos(\theta - C)}{\sin C}$$

$$\text{Therefore } \cos \theta \cot B - \sin \theta = \cos \theta \cot C + \sin \theta$$

$$\text{Therefore } \tan \theta = \frac{1}{2} (\cot B - \cot C)$$

$$\therefore \theta = \tan^{-1} \left\{ \frac{1}{2} (\cot B - \cot C) \right\}$$

A PROJECT REPORT
ON

"Study Of Propositional Logic"

PAPER CODE:- MAT-HE-6086

IN PARTIAL REQUIREMENT OF FULFILMENT OF THE DEGREE
OF BACHELOR OF SCIENCE IN MATHEMATICS, 2023



Submitted by:-

JAHIRUL ISLAM

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Evaluation committee

1. Chairman / *External*
2. H.O.D.
3. Supervisor

signature

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(A.B. Siddique).
Atiqur Rahman.
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Shaukat
24/05/23

CERTIFICATE

This is certified that Jahirul Islam, A student of B.Sc. 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) has carried out this project report entitled "PROPOSITIONAL LOGIC" by himself under my supervision. He has fulfilled all the requirements for the preparation of project report for the 4th paper of B.Sc.(CBCS) 6th semester examination, 2023

He worked very hard to prepare this project report.

I wish him very success in his life.

Date: 23/15/2023

Place: Nagbua


Supervisor

CERTIFICATE

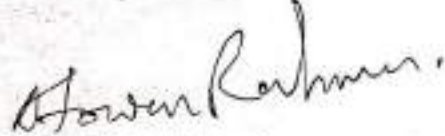
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Date: 22/05/2023

place : Nagarkurnool

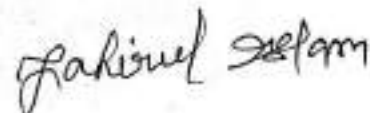

Head of the Department
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ACKNOWLEDGEMENT

This project report entitled " PROPOSITIONAL LOGIC" is intended for drawing attention of mathematical applications in the field of Algebra

My sincere thanks to professor Dr.Atower Rahman,Head of the department of mathematics of B.P. Chaliha College,Nagarbera for his guidance and providing necessary facilities to carry out my project report.

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PROPOSITIONAL LOGIC

INTRODUCTION.

Logic is the basis of all mathematical reasoning and of all automated reasoning. The rules of logic specify the meaning of mathematical statement. These rules help us understanding and reason with a statement such as -

there exist x such that $x \neq a^2 + b^2$ where $x, a, b \in \mathbb{Z}$. which in simple english means "there exists an integer that is not the sum of two squares." The logic provides, basic to many areas of computer science such as digital logic designing and computability. "propositional logic also known as sentential logic and statement logic" is the branch of logic that studies ways of joining and modifying entire propositions, statement or sentence to form more complicated propositions, sentences or statements as well as the logical relationships and properties that are derived.

1.1 BASIC CONCEPT OF PROPOSITION OR STATEMENT.

1.1.1 PROPOSITION (OR STATEMENT)

In mathematics, one deals with a large number of sentences which are strings of words and symbols having precise meanings. All such sentences are classified as being true or false and they are called statements. Thus, a statement is either true or false; never both or in between. For examples 'Milk is white' and 'Assam is the capital of India'. The first sentence is true and is a statement, while the second statement is false. Basically, a statement expresses a judgement or opinion.

The truth value of proposition is true and it denoted by 'T', on the other hand the false value of a proposition is false and it is denoted by 'F'.

for example.

- (i) 5 is a prime number.
- (ii) $7+3=10$
- (iii) $5+3=9$
- (iv) Moscow is the capital of china.

All the above sentences are proposition or statement where the first and second statement are true and third and fourth are false.

The sentence: "The sentence I am reading is false." it is not a statement because it can be neither true or false.

A sentence can be a statement even if you do not know decisively whether it is true or false, or even if its being true or false depends on the context. For example—

- (i) Every even integer ≥ 4 is a sum of two primes.
- (ii) $x + 3 \geq 7$.
- (iii) The number of students in the class is fifty.

However, it is not yet known whether sentence (i) is true or false. The sentences (ii) and (iii) are context specific; they are true or false depending on the situations. (ii) is true if $x \geq 4$ but false if $x = 3$. Nevertheless, the above sentences are statements.

NOTE: It is customary to represent simple statement by $p, q, r, \dots$ known as propositional variable. Propositional variable can only assume two values true or false. These are two propositional constants. T and F that represent true or false respectively.

1.1.2. COMPOUND PROPOSITION

To represent propositions, propositional variables are used. By convention, these variables are represented by small alphabets such as the area of logic which deals with propositions is called propositional calculus or propositional logic. Propositions constructed using one or more propositions are called compound statements.

consider the statement: "The boy is intelligent and handsome." Notice that it consists of two statements joined by "and." the statement may be read as "The boy is intelligent and the boy is handsome."

A statement of this kind is called a conjunction of statements.

1.1.3 CONNECTIVES.

The operational symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ are called connectives. From now on we shall assume that the letters $p, q, r, \dots$ with or without subscripts, denoted variable over the set of statements. We call them statement letters.

The following are used to represent connectives -

Symbols used	connective word	Nature of the compound statement formed by connectives	symbolic form
$\neg$, —	not	Negation	$\neg p$
$\wedge$	and	conjunction	$p \wedge q$
$\vee$	or	disjunction	$p \vee q$
$\Rightarrow$, $\rightarrow$	if... then	Implication	$p \Rightarrow q$
$\Leftrightarrow$, $\leftrightarrow$	if and only if	equivalence	$p \Leftrightarrow q$
$\oplus$, $\otimes$	XOR	'Exclusive OR' (OR XOR)	$p \oplus q$
$\downarrow$	NOR	Negation of disjunction	$p \downarrow q$
$\uparrow$	NAND	Negation of conjunction	$p \uparrow q$

1.2 OPERATION ON STATEMENT

The operations are truth functional in the sense of that truth value of the new statement are determined by the truth value of the component statement.

1.2.1 NEGATION

For any given statement P , there is another statement which is defined to be the statement that is true when P is false, and false when P is true. This statement is called the negation of P and is denoted by $\neg P$ or $\sim P$, which when translated to simple english means 'It is not the case that P or simply not P '. The truth value of $\neg P$ is the opposite of the truth value of P .

The negation of the statement " $5+3=9$ " is $5+3 \neq 9$, and not the statement " $5+3=8$ ".

The truth table of $\neg P$ is —

P	$\neg P$
T	F
F	T

1.2.2 CONJUNCTION

For any two propositions p and q , their conjunction is denoted by $p \wedge q$, which means 'p and q'. The conjunction $p \wedge q$ is true when both p and q are true, otherwise false.

The truth table of $p \wedge q$ is-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

1.2.3 DISJUNCTION

For any two propositions p and q their disjunction is denoted by $p \vee q$, which means 'p or q'. The disjunction $p \vee q$ is true when either p or q is true otherwise false. The truth table of $p \vee q$ is-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

1.2.4. CONDITIONAL (OR IMPLICATION)

For any two propositions p and q , the statement 'if p then q ' is called a conditional $p \rightarrow q$. p is called the hypothesis and q is called the conclusion or consequence. The implication is false when p is true and q is false otherwise it is true.

The truth table of $p \rightarrow q$ is -

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

1.2.5: BICONDITIONAL (OR- BI-IMPLICATION)

The operation 'bi-conditional' combines two statements p, q to form a new statement 'p if and only if q ' denoted by $p \leftrightarrow q$. $p \leftrightarrow q$ has the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$. The implication is true when p and q have same truth value and is false otherwise.

The truth table of $p \leftrightarrow q$ is—

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

1.2.6 "EXCLUSIVE OR" OR "XOR"

For any two propositions p and q their exclusive or is denoted by $p \oplus q$ which means "either p or q but not both". The exclusive or $p \oplus q$ is true when either p or q is true and false when both are true or false.

The truth table of $p \oplus q$ is—

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

1.2.7. DERIVED CONNECTIVES.

NOR:

It means negation of disjunction of two statements. NOR of p and q is true when both p and q are false. Denoted by $p \downarrow q$ its truth table is —

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

NAND:

It means negation of conjunction of two statements. NAND of proposition p and q is false when both p and q are true, otherwise true. It is denoted by $p \uparrow q$ and $p \uparrow \equiv \neg(p \wedge q)$

Its truth table is as follows —

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

1.2.8 STATEMENT FORMULA

A statement formula is defined recursively as follows -

- (i) Any statement letter is a statement formula.
- (ii) If A and B are statement formula then the expression $\neg A$, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B$ are also statement formula.
- (iii) Only those expression are statement formula which are considered by using (i) and (ii).

1.2.9 TRUTH FUNCTION.

For every assignment of truth values T or F to the statement letters occurring in a statement formula there corresponds by virtue of the truth table for the connectives, a truth table for the statement formula. Thus each statement formula containing n distinct statement letter determines a function called truth function i.e., a function from $\{T, F\}^n$ into $\{T, F\}$, which can be graphically represented by a truth table for the statement formula.

1.2.10 TAUTOLOGY

A statement formula is called a tautology if in its truth table the column under the statement formula contains only T 's i.e. if its truth functions

takes always the value T. We generally use the notation 'F' to indicate that A is a tautology. ('F' is known as K-leen's symbol).

for example: $\neg(p \wedge q) \vee q$ is a tautology.

1.2.11 CONTRADICTION

A statement formula is called a contradiction if in its truth table, the column under the formula containing only F's i.e; if its truth function takes always the value F's.

for example: $p \wedge (q \wedge \neg p)$ is a contradiction.

NOTE: A statement formula which is neither a contradiction nor a tautology is called contingent.

1.2.12 LOGICAL CONSEQUENCE.

If $A \rightarrow B$ is a tautology then A is said to logically imply B, or alternatively, B is said to be logical consequence of A. for example: $p \wedge (p \rightarrow q) \rightarrow q$ is a tautology.

1.2.13 LOGICAL EQUIVALENCE:

If $A \leftrightarrow B$ is a tautology then A and B are said to be logically equivalent and we denote this fact by $A \equiv B$. observe that $A \equiv B$ if and only if they have the same truth table.

for example: $p \rightarrow q \equiv \neg p \vee q$.

1.3 LAWS OF PROPOSITIONAL LOGIC

By truth tables, using the principle of substitution we can easily verify that the following laws are valid in the set of statement formulas. Let A, B, C be any three statement formula, then

(i). IDEMPONENT LAW:

$$A \vee A \equiv A$$

$$A \wedge A \equiv A$$

(ii). ASSOCIATIVE LAW:

$$(A \vee B) \vee C \equiv A \vee (B \vee C)$$

$$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$$

(iii). COMMUTATIVE LAW:

$$A \vee B \equiv B \vee A$$

$$A \wedge B \equiv B \wedge A$$

(iv). DISTRIBUTION LAW:

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

(v). DE-MORGAN'S LAW:

$$\neg (A \vee B) \equiv \neg A \wedge \neg B$$

$$\neg (A \wedge B) \equiv \neg A \vee \neg B$$

(vi). ABSORPTION LAW:

$$A \vee (A \wedge B) = A$$

$$A \wedge (A \vee B) = A$$

(vii). CONTRAPOSITIVE LAW:

$$A \rightarrow B \equiv \neg B \rightarrow \neg A$$

(viii). IDENTITY LAW:

$$A \vee F \equiv A$$

$$A \vee T \equiv T$$

$$A \wedge T \equiv A$$

$$A \wedge F \equiv F$$

where T and F denotes respectively a tautology and a contradiction.

(ix). COMPLEMENT LAWS:

$$A \vee \neg A \equiv T$$

$$A \wedge \neg A \equiv F$$

$$\neg T \equiv F$$

$$\neg F \equiv T$$

(x). INVOLUTION LAWS:

$$\neg(\neg p) \equiv p.$$

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CERTIFICATE

Certified that TASLIMA NASRIN, a student of B.Sc. final year of department of Zoology of Bimala Prasad Chaliha College, Nagarbera has worked out his field project work, entitled studies on "Bird nest building and Bird Behavior" in Nagarbera Area (Nagarbera) under my supervision.

Sonali
16/5/23
(Nupam Kr. Goswami)
Sign. of Supervisor,

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INTRODUCTION:

The Brahmaputra drainage system in North-East India is one of the largest hydrographic basin. Southeast Asia and sustains very rich and diverse aquatic gene pool, particularly fishes and such as the region is featured among the global hotspot of fresh water fish biodiversity. All north eastern states have handful resources of fishes as well as other aquatic species in terms of many rivers with their tributary's streams, rivulets, wetlands, lakes, ponds, tank etc. Assam has vast and varied fresh water resources and it being regarded as one of the richest spots of biodiversity in India.

Fish invariably one of the most important biotic components of an aquatic ecosystem which apart from forming protein rich food source for human beings, also act as a good bio indicator of a water body. The Northeastern region of India is one of the hot spots of fresh water fish biodiversity in the world. However, the rich biodiversity of the freshwater fish of the India has been rapidly declining over the year due to excessive human activities and as well as another environment factor, the loss of biodiversity and its effect are predicted to be greater for aquatic ecosystems than for terrestrial ecosystems Sala et al. (2000).

CONCLUSION:

In conclusion documentation of biodiversity of fish is one of the most important aspects of study from ichthyological point of view. No fishing should be allowed during spawning season especially with mosquito nets for sustainable ecosystem management as well as to sustain the identified fish fauna in the beel. People should be aware about the threatening factors responsible for declining of fish fauna of the river .

CERTIFICATE

Certified that **RASHIDA AHMED**, a student of B.Sc. final year of department of Zoology of Bimala Prasad Chaliha College, Nagarbera has worked out his field project work, entitled studies on "Bird nest building and Bird Behavior" in Nagarbera Area (Nagarbera) under my supervision.


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(Dr. Purnima K. Goswami)
Signature of Supervisor

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CERTIFICATE

Certified that JAHANGIR AZAD, a student of B.Sc. final year of department of Zoology of Bimala Prasad Chaliha College, Nagarbera has worked out his field project work, entitled studies on "Bird nest building and Bird Behavior" in Nagarbera Area (Nagarbera) under my supervision.

Sd/-
15/05/2023
Dr. Papan Kumar (Sahasrini)
Signature

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INTRODUCTION

Nest building is a taxonomically widespread activity, with birds, mammals, reptiles, fish, and insects all constructing nests of some description in which to lay eggs and/or raise offspring. There is a huge amount of variation in nest design across taxa, with nests varying from underground burrows dug by mammals, minimal nest scrapes on the ground in which game birds lay their eggs, the craters constructed by fish on the bed of water bodies, the huge mounds constructed by termites through to the cup-shaped nests of song birds in trees and bushes. Even within taxa, there is a great deal of variation in nest design and in birds, nests range from the small but elaborate cup-shaped nests built by passerine birds through to the huge mounds built by megapodes.

Nest design varies considerably between and even within taxa, yet all nests have the same basic, minimal, function which is to provide a receptacle in which animals can lay their eggs and/or raise their developing offspring. This rather simplistic view of nests has generally prevailed over the years, and studies examining the function of birds' nests have been scarce, particularly when compared to other stages of reproduction. Illustratively, a study of six commonly studied nestbox-breeding passerine birds showed that fewer than 6% of 676 published studies reported any aspect of nest characteristics, despite researchers using nestbox-breeding birds as model systems due to the ease with which their reproductive parameters can be quantified. This oversight is unfortunate as there is considerable evidence that nests are sophisticated structures that require considerable cognitive abilities to construct. Fortunately, our understanding of the design and function of birds' nests has increased considerably in recent years, and here, we review the functions of birds' nests. We begin by examining the



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Discussion

Overall, our meta-analysis did not detect any strong relationship between urbanization and nest mortality. However, predation rates of artificial nests and of natural nests showed opposite trends in their relationships with urbanization: namely, the chance of natural nests to fail tended to decrease (i.e., the odds of survival of natural nests increased by 33%) with increasing urbanization, whereas the trend in artificial nests was significantly different, and in the opposite direction (i.e., the odds of survival was about 35% lower in more urbanized habitats). None of our considered moderators, ecological or methodological, explained the variation in effect sizes consistently in either study type, although our sensitivity analyses suggest nest openness may have an effect (Table S11, Supplementary Discussion S3.1).

The weak positive trend for a correlation between nest survival and urbanization in natural nests is in line with the “predator-safe zone hypothesis” (Gering and Blair, 1999; Ryder et al., 2010), which assigns the lower predation risk in cities as a principal reason of why certain prey species can thrive in urban habitats. Lower predation rates can be the result of low abundance of nest predators (Møller, 2012). However, many potential nest predator species are found in higher abundances in cities than in the surrounding natural habitats (Jokimäki and Huhta, 2000; Haskell et al., 2001). This apparent contradiction is called the “nest predation paradox” (Fischer et al., 2012). This paradox might be resolved

in several ways. First, some prey species are often extremely abundant and/or found in higher densities in more urban habitats (McKinney, 2006). Thus, despite the high absolute numbers of nest predators, their relative abundance to prey can still be low compared to natural habitats (Fischer et al., 2012). Second, urbanization changes predator species composition (Rodewald and Kearns, 2011). This shift can mean a decrease in specialized ("strong") nest predators and an increase in opportunistic ("weak") nest predators (Stracey, 2011). For example, snakes, which are often specialized nest predators, are less abundant in cities than in rural areas (Patten and Bolger, 2003). In contrast, house cats, which are both fed by humans and hunt for live prey, and thus are mostly opportunistic nest predators, become more abundant with housing density (Sims et al., 2008), although they might be less common (or less often outdoors) in city centers compared to suburbs. Third, potential predators may specialize on different prey in urban than in rural areas as the most abundant prey species might differ between them, which can relax predation pressure on less abundant species (Fischer et al., 2012). Alternatively, in the case of omnivorous predator species, the shift can be toward anthropogenic food sources that are easier to access, which can relax the actual predation pressure on all prey species (Rodewald et al., 2011). Finally, higher abundance of nest predators in urban habitats can facilitate local adaptation in the behavior of their prey. As high predation pressure in cities should eliminate those individuals that could not effectively defend their nests from predators (either by hiding their nests or

actively mobbing predators), the current urban prey population is better at nest defense and thus has higher survival rates than rural prey (Stracey, 2011).

In contrast to natural nests, we found a decreasing trend in the survival of artificial nests with increasing urbanization. This result is in line with a meta-analysis which revealed more predation on artificial nests with increasing fragmentation of forest habitats (Hartley and Hunter, 1998). Also, similar to our findings, a number of studies showed that predation rates in artificial nests often do not reflect those observed in natural nests (Haskell, 1995; Weidinger, 2001; Moore and Robinson, 2004; Robinson et al., 2005), although one study that used artificial and natural nests in the same conditions did not find significant difference between the two (Blair, 2004). The discrepancy between artificial and natural nests may be explained by several different mechanisms. First, our sensitivity analyses on natural nests indicated that predation rates of cavity nests and open nests change differently with urbanization (Table S11). Namely, cavity nests are predated significantly less in urban than in rural habitats, while open nests show no such habitat difference. Since the majority of artificial nest studies were conducted on open nests, their results are more comparable to those from natural open nests than to those from hole-like nests (which are also likely to be located high above the ground and exposed to different conditions). Second, as mentioned above, it is hypothesized that local adaptation may make urban bird parents better at nest defense behavior than their non-urban conspecifics (Stracey, 2011). As artificial nests are not defended

by parents, they may be more likely to be depredated in urban habitats than natural nests. Third, the local adaptation hypothesis also suggests that urban birds are better at hiding their nests from predators (Stracey, 2011). As the locations of artificial nests are chosen by the experimenters, rather than the birds themselves, nests may be placed in more conspicuous places in urban habitats, thus predators can find them more easily, resulting in a higher predation rate. Fourth, although artificial nests try to emulate natural nests as much as possible, they might still be perceived by predators as novel compared to natural nests. It has been hypothesized that urban animals show less food neophobia (i.e., are more likely to accept novel food sources) than their rural conspecifics (Sol et al., 2011), and thus non-urban predators may be aversive toward artificial nests. Fifth, some discrepancy between the results of natural vs. artificial nest studies may come from the fact that partially predated nests were counted as survived in most natural nest studies, whereas they were counted as predated in artificial nest studies. Partial predation may be more common in cities where the predator is more likely to be interrupted by human disturbance while feeding on a nest. Finally, we cannot exclude the possibility that the real predation pressure on natural nests is higher in cities than in rural habitats, as studies on artificial nests suggest, but the effect is masked by sampling bias. Urban natural nests that were not concealed or defended well-enough may already have been predated before the researchers found them, and thus not included in the sample, while this might not be the case in rural habitats. The

resulting sampling bias may lead to a seemingly higher predation rate in rural compared to urban habitats.

In conclusion, our results show that natural nests tend to be less predated in urban habitats than in rural habitats, but this trend is not reflected by studies on artificial nests. We have several recommendations for future studies addressing the relationship of urbanization and nest predation. First, we suggest that the cause of nest failure should be identified as precisely as possible, because both predation and other forms of mortality (weather, nest abandonment, vandalism by humans) can vary with urbanization, affecting the overall nest survival. Second, to identify whether variation is due to differences between species, multiple populations of the same species should be studied, and studies should preferably include data from multiple species. Third, data from the currently available literature is not sufficient for more sophisticated analyses, such as testing multiple biological and ecological factors in the same model and possible interactions between them, therefore more studies, with more balanced design for ecological characteristics such as nest height and nest openness, are required. Finally, as patterns derived from studies using artificial nests often do not qualitatively reflect those derived from natural nests, researchers should perform studies where artificial and natural nests with similar characteristics are monitored within the same area (e.g., Blair, 2004), and investigate sampling bias. Such validation is important to draw firm conclusions regarding the level of predation in urban vs. rural areas in future studies.

The reasons for the decline of the sparrow population are loss of habitat due to rapid urbanisation, diminishing ecological resources for sustenance, high levels of pollution and emissions from microwave towers.

প্ৰকল্প (PROJECT)

SEC COURSE

শিৰোনাম : ভাৰতবৰ্ষৰ বিৰুদ্ধে গৃহীতকৰণ প্ৰক্ৰিয়া

লেখক : বিজ্ঞানীতা কলিতা

শ্ৰেণী : তৃতীয় মাধ্যমিক

বোল নং : ৫০

Certificate

This is to certify that Miss Bijusmita Kalita has worked under my guidance and supervision for her project work on SEC COURSE, which is being submitted to the Dept. of Political Science.

Date : 28/05/2023

Place : Nagambena



(Dr. Diganta Kalita)
Assl. - professor

বিষয়সূচী
(content)

পৃষ্ঠা নং

আবহুত

1

বিবেকীয় পৰিচয়

2

বিবেকীয় গৃহীতকৰণ প্ৰক্ৰিয়া

5

সামৰণ

8

১. ভাষা অর্থাৎ বিবিধ পুঁজিবসন প্রসিদ্ধি আন্দোলন
কৰা।

উত্তৰ :-

আবস্থান :

অন্যতঃ স্বাধীনত সাংবিধানিক স্বাধীনত
মৌলিক নীতিবোৰ লিপিবদ্ধ থাকে। সাংবিধানিক এখন
মোড়াদি সেই নীতি আন্দোলনৰ স্বাভাৱত ৰূপ দিয়া
হয়। তাৰতীয়া সাংবিধানিক অতিৰিক্তে গুৰুত্বপূৰ্ণ কাৰ্য্যদি-
বোৰেই হ'ল অহিন প্ৰনয়ন কৰা। সাংবিধানিক তাৰতীয়া
অহিন প্ৰনয়নৰ সকলো দক্ষিণ কেন্দ্ৰীয় অহিন সত্তা
আৰু স্বাভাৱিক অহিন সত্তাৰ হাতত অৰ্পন কৰিছে।
কেন্দ্ৰীয় সৰ্ব্বমুখী কেন্দ্ৰীয় তালিকাভুক্ত বিষয়সমূহী আৰু
প্ৰয়োজনবোধে স্বাভাৱিক তালিকাভুক্ত বিষয়সমূহীৰ ওপৰতো
অহিন প্ৰনয়ন কৰিব পাৰে। তাৰোপৰি সন্মতী সূচীৰ
বিষয় আৰু অন্তৰ্গত সন্মতীৰ বিষয়ৰ ওপৰতো কেন্দ্ৰীয়
সৰ্ব্বমুখী অহিন প্ৰনয়ন কৰিব পাৰে।

② বিবেকীয়কৰ পৰিচয় :-

সংসদৰ অনুমোদনৰ কাৰণে আইন
যি প্ৰস্তাৱ উত্থাপন কৰা হয় তাক বিবেকীয়ক বা বিল বোলে
গতিকে বিবেকীয়ক হ'ল সংসদত প্ৰস্তাৱিত আইন এখনৰ
মতৰা অৰ্থাৎ আইন পাতুলিপিৰ বিবেকীয়ক বা বিলকে
হয়। বিল এখনক সাংবিধানিক প্ৰক্ৰিয়া অনুসৰি গৃহীত
হোৱাৰ পিছতহে আইনত পৰিণত হয়। এইখন বিবেকীয়ক
এটা নিৰ্দিষ্ট শিৰোনামেৰে আৰম্ভ হয়। বিবেকীয়ক
এখনত এটা প্ৰস্তাৱনা আৰু কাৰ্যকৰণৰ সূত্ৰ থাকে।
উদাহৰি বিবেকীয়ক এখনৰ প্ৰয়োগযোগ্যতাৰ পৰিমাণ দিয়া
হয় আৰু কিছুক্ষেত্ৰত ই আইনত পৰিণত হোৱাৰ তালিকা
নিৰ্দিষ্টকৈ দিয়া হয়।

কেন্দ্ৰীয় বিধানসভা বা সংসদৰ
আটাইতকৈ গুৰুত্বপূৰ্ণ কাৰ্যদোৱেই হৈছে দেশৰ প্ৰশাসন
কাৰ্য পৰিচালনা কৰাৰ ক্ষেত্ৰত প্ৰয়োজন হোৱা আইন
সমূহ প্ৰণয়ন কৰা। যাৰ আৰম্ভণি দেশৰ কাৰ্যপালিকাই
প্ৰশাসন কাৰ্য সুচাৰুৰূপে পৰিচালনা কৰাৰ প্ৰয়াস
কৰে। এই প্ৰশাসন কাৰ্য সু-সুখমাতে পৰিচালনা
কৰাৰ প্ৰয়াস কৰে এই প্ৰশাসনকাৰ্য সু-সুখমাতে
পৰিচালনা কৰাৰ বাবে প্ৰয়োজন হোৱা এই আইন
সমূহক আৰম্ভণিতে এক মতৰাবিধি অথবা প্ৰস্তাৱ
আকাৰে অনুমোদনৰ বাবে সংসদ উত্থাপন কৰা হয়।

সম্মত অর্হিতৰ এই পদ্ধতিবিধি বা প্ৰভাৱসমূহকে স্ৱাৰ্থাৰ্থন
 বিন বা বিৰ্বেয়ক হিচাপে বিবেচনা কৰা হয়। এই বিন
 বা বিৰ্বেয়কত নিৰ্বাচিত প্ৰক্ৰিয়াৰ সাজেৰে সাজেৰে অনু-
 য়োজন জনালে আৰু স্বাৰ্থপতিয়ে যদি তাত সন্মতি প্ৰদান
 কৰে তেন্তে ই অৰ্হিতলৈ অপাত্তিৰ্ত্ত হয়। এইবোৰ হৈছে
 কিছুমান বৈৰ্লিক প্ৰভাৱ যিবলক বিন অথবা বিৰ্-
 য়কৰ কথত সাজেৰে সাজেৰে উত্থাপন কৰা হয়।

স্ৱাৰ্থাৰ্থনতে বিন অথবা বিৰ্বেয়কসমূহক
 বহুলভাৱে চৰকাৰী বিৰ্বেয়ক আৰু বেচৰকাৰী
 বিৰ্বেয়ক দুই ভাগত ভগাব পাৰি।

১) চৰকাৰী বিৰ্বেয়ক :

বিৰ্ৱালিত প্ৰক্ৰিয়াৰ বাবে সাজেৰে
 সাজেৰে দুই বৰ্ণনৰ বিৰ্বেয়ক উত্থাপন কৰা হয়। যেনে
 চৰকাৰী বিৰ্বেয়ক আৰু বেচৰকাৰী
 বিৰ্বেয়ক। চৰকাৰী বিৰ্বেয়ক সাজেৰে যিকোনো সাজেৰে
 উত্থাপন কৰিব পাৰি। এনে বিৰ্বেয়ক সাজেৰে উত্থাপন
 কৰে যিয়ে চৰকাৰৰ প্ৰতি সাজেৰে সাজেৰে সাজেৰে
 সাজেৰে সাজেৰে এনে বিৰ্বেয়ক সাজেৰে সাজেৰে

কৰাৰ পাছতহে এই বিবেকীয়ক সঙ্গূহে আইনত পৰিচিত
হয়।

স্বাস্থ্যকৰ এখন সন্দনত বিবেকীয়ক এখন গৃহীত
হোৱাৰ পিছত আনখন সন্দনলৈ অনুমোদনৰ বাবে
পঠিওৱা হয়। আনখন সন্দনতো বিবেকীয়কখন
বিভিন্ন পৰ্যায়ৰ স্তৰেৰে আলোচনা কৰা হয়।
শেষত আনখন সন্দনে বিবেকীয়কখন —

- ① কোনো স্বাস্থ্যকৰ নকৰাকৈ বিবেকীয়কখন
প্রথম সন্দনে দিয়া বৰ্ণনে গ্রহণ কৰিব পাৰে,
- ② বিবেকীয়কখন সঙ্গূহৰ নাকচ কৰিব পাৰে,
- ③ বিবেকীয়কখন স্বাস্থ্যকৰ নকৰাকৈ কৰি পুনৰ
বিবেচনাৰ বাবে প্রথম সন্দনলৈ ঘূৰাই পঠিয়াব
পাৰে।
- ④ কোনো সিদ্ধান্ত নোলোৱাকৈ সঙ্গূহ নিষ্ক্ৰিয়তা
পেনাই ৰাখিব পাৰে।
- ⑤ স্বাস্থ্যকৰ :-
আপুনি উল্লেখ কৰাৰ দৰে তাত্ত্বিক

স্বাধীনতা উন্নয়ন সনদ বিল বীক্ষা করা হয়। কিন্তু সারা
বর্ষে স্বাধীনতা সনদ লোকসভার হাতে মথিত
সুযোগ অর্জন করিছে। যদিও স্বাধীনতা সনদ খুঁ
করিব নোবাৰি।

ভাৰতৰ দৰে এখন গণতান্ত্ৰিক দেশত
বিল বা বিৰোধীক এক উল্লেখনীয় বিষয়। এই বিল
বিহীন প্ৰক্ৰিয়াৰে স্বাভাৱে অহিনে পৰিণত হয়, দেশ
সুৰক্ষাৰে চলাবলৈ মথিত স্বায় কৰে।

৩) প্ৰজ্ঞাপন পুৰি :-

ছবুৰ ড'ক্টৰ

ৰায়, সুদৰ্শন, ২০৪০ স্বাধীনতা কৰ্ম আৰু দণ্ড

দেৱাজিৎ অনুশীলন ইণ্টিনিমন বুক পাব্লিকেশন।

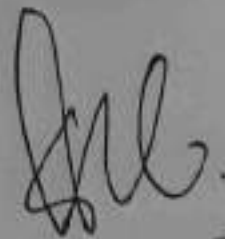
— X —

Certificate

This is to certify that Miss Trishna Kalita has worked under my Guidance and supervision for her project work on SEC COURSE, which is being submitted to the Dept. of Political Science.

Date : 28/05/2023

Place : Nagaon



(Dr. Dagantha Kalita)
Asst. Professor

বিষয়সূচী
(Content)

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১. ~~৪.১.~~ ভাৰতবৰ্ষত বিৰ্বৈয়ক গৃহিত কৰণ প্ৰক্ৰিয়াটো
আলোচনা কৰা ?

~~উত্তৰ :-~~ ভাৰতবৰ্ষত বিৰ্বৈয়ক গৃহিত কৰণ প্ৰক্ৰিয়া
টো তলত আলোচনা কৰা হ'ল -

আবস্থান :- কেন্দ্ৰীয় বিধানসভা বা সংসদৰ
আটাইতকৈ গুৰুত্বপূৰ্ণ কামটোকেই হৈছে দেশৰ প্ৰশাসন
কাম পৰিচালনা কৰাৰ ক্ষেত্ৰত প্ৰয়োজন হোৱা আইন
সমূহ প্ৰণয়ন কৰা। যাৰ আৰম্ভণি দেশৰ কাম-
পালিকাৰ প্ৰশাসনকাম সুচাৰুৰূপে পৰিচালনা
কৰাৰ প্ৰয়াস কৰে। এই প্ৰশাসনকাম সু-শৃংখ
লাৰে পৰিচালনা কৰিবৰ বাবে প্ৰয়োজন হোৱা এই
আইনসমূহক আবস্থানতে এক খণ্ডবিধি অথবা
প্ৰস্তাৱ আকাৰে অনুমোদনৰ বাবে সংসদে
উত্থাপন কৰা হয়। অন্তৰ্গত আইনৰ এই খণ্ডবিধি
বা প্ৰস্তাৱসমূহক আৰম্ভণিতে বিল বা বিৰ্বৈয়ক
হিচাপে বিবেচনা কৰা হয়। এই বিল বা বিৰ্বৈয়কত

নিৰ্ধাৰিত প্ৰক্ৰিয়াত সাদৰ অনুমোদন
 জনালে আৰু বাস্তৱপাতিয়ে যদি তাত সন্মতি প্ৰদান
 কৰে তেন্তে ই আইনলৈ ৰূপান্তৰিত হয়। এইবোৰ
 হৈছে কিছুমান বৈধানিক প্ৰস্তাৱ, যিবোৰক বিল
 অথবা বিৰ্ণয়কৰ ৰূপত সাদৰ সাদৰ সাদৰ প্ৰস্তাৱত উত্থাপন
 কৰা হয়। অৱশ্যেই এই আইনসমূহক প্ৰাৰ্থন
 সন্নিৱিৰ্ণিত উল্লেখ কৰা হয়। আইন প্ৰণয়নৰ
 বাবে সাদৰত উত্থাপন কৰা এই বিল বা বিৰ্ণয়ক
 সমূহ বিভিন্ন প্ৰকাৰৰ হ'ব পাৰে।

বিৰ্ণয়কৰ দৰিদ্ৰ :- সাদৰ অনুমোদনৰ কাৰণে
 আইনৰ যি প্ৰস্তাৱ উত্থাপন কৰা হয় তাক বিৰ্ণয়-
 ক বা বিল বোলে। গতিকে বিৰ্ণয়ক হ'ল সাদৰত
 প্ৰস্তাৱিত আইন উত্থাপনৰ প্ৰচাৰ। অৰ্থাৎ আইনৰ
 প্ৰাৰ্থনালৈ বিৰ্ণয়ক বা বিল বোলা হয়। বিৰ্ণয়ক
 উত্থাপন সাংবিধানিক প্ৰক্ৰিয়া অনুসৰি গৃহীত হোৱাৰ
 পিছতহে আইনত পৰিণত হয়।

সাধাৰণতে সাদৰত উত্থাপন কৰা বিৰ্ণয়ক
 দুই ভাগত বিভক্ত কৰা হয়। যেনে - ক) চৰকাৰী

বিবেক আৰু ৩) বেচকাৰী বিবেক ।

সেইদৰে চৰকাৰী বিবেকক আকৌ — ① আবিষ্কাৰ বিবেক, অৰ্থ অজ্ঞাত বিবেক আৰু বিত্ত অজ্ঞাত বিবেক আদি ভাগত ভাগ কৰা হয় । এইখিনিতে এটা কথা উল্লেখযোগ্য যে বহুক্ষেত্ৰত অৰ্থ অজ্ঞাত বিবেক আৰু বিত্ত অজ্ঞাত বিবেকক একেটা অৰ্থত ব্যৱহাৰ কৰা দেখা যায় । কিন্তু প্ৰকৃতপক্ষে এই দুয়োটা বিবেকৰ মাজত কিছুমান পাৰ্থক্য আছে ।

অৰ্থ অজ্ঞাত বিবেক আৰু বিত্ত বিবেকৰ মাজৰ পাৰ্থক্যসমূহ হ'ল —

- i) অৰ্থ অজ্ঞাত বিবেক সম্পৰ্কে অংবিধানৰ ১১০ নং অনুচ্ছেদত আলোচনা কৰা হৈছে । আনহাতে বিত্ত বিবেক সম্পৰ্কে অংবিধানৰ ১১৭ নং অনুচ্ছেদত আলোচনা কৰা হৈছে ।
- ii) অৰ্থ অজ্ঞাত বিবেক উত্থাপনৰ ক্ষেত্ৰত বাণিজ্য দপ্তৰ আণ্ডীনা অনুমোদনৰ প্ৰয়োজন হয় । কিন্তু বিত্ত বিবেক উত্থাপনৰ ক্ষেত্ৰত এনে ধৰণৰ

অনুশীলনৰ প্ৰয়োজন নহয় ।

(iii) অৰ্থ অনজ্ঞাত বিৰ্য্যকৰ ক্ষেত্ৰত লোকজ্ঞাৰ অৰ্য্যকৰ্হ প্ৰমাণ দাও দিব লাগে । কিন্তু বিত্ত বিত্ত বিৰ্য্যকৰ ক্ষেত্ৰত এনে প্ৰমাণ প্ৰদৰ্শন প্ৰয়োজন নহয় ।

(iv) অৰ্থ অনজ্ঞাত বিৰ্য্যক অকল স্বাত্ৰ লোকজ্ঞাতৰে উদ্ভাৱন কৰিব পাৰি । কিন্তু বিত্ত বিত্ত বিৰ্য্যক লোকজ্ঞা বা স্বাত্ৰজ্ঞাত উদ্ভাৱন কৰিব পাৰি ।

এইদৰে দেখা যায় যে অৰ্থ অনজ্ঞাত বিৰ্য্যক আৰু বিত্ত বিৰ্য্যকৰ স্বাত্ৰত হোৱা প্ৰমাণ দাৰ্শন্য আছে । এই দাৰ্শন্যসমূহ অৱগত কৰা আৰু ছাত্ৰ-ছাত্ৰীসকলক বিৰ্য্যকৰ প্ৰকাৰ সম্বন্ধে স্পষ্ট জ্ঞান দিয়াৰ কাৰণে এই অৰ্য্যকত প্ৰয়াস কৰা হৈছে ।

এই প্ৰক্ৰিয়াতে আমি চৰকাৰী আৰু

বিবেচনাৰ বাবে তেওঁৰ বানীৰ সৈতে অসঙ্গতলৈ
 ওলোতাৰ্হ পাচিয়াৰ পাৰে। এনে ক্ষেত্ৰত অসঙ্গত
 পুনৰ্বিবেচনা কৰি বিৰোধপন আকৌ বাদ্ৰুপতিলৈ
 প্ৰেৰণ কৰিলে বাদ্ৰুপতিয়ে তাত অম্মতি প্ৰদান
 কৰিবলৈ বাৰ্য্য। কোনো বিৰোধকত বাদ্ৰুপতিয়ে
 দ্বাৰ্য্যৰ দান কৰাৰ পিছত ই আইনত পৰিণত
 হয় আৰু চৰকাৰে নিৰ্দ্ধাৰণ কৰা তাৰিখৰ
 পৰা সেই আইন বলৱৎ কৰা হয়। আইনত
 পৰিণত হোৱাৰ পাছত ইয়াক চৰকাৰী বাদ্ৰু-
 পত্ৰত প্ৰকাশ কৰা হয়।

আগ্ৰবানি :- যিখৈ নষ্টক, অন্যান্য আৰ্য্যৰণ
 বিৰোধক বা চৰকাৰী বিৰোধকৰ দৰে একেই
 প্ৰক্ৰিয়া আৰু নীতি নিয়মৰ আধোদি অসঙ্গতৰ
 টোৱে অদৰে চৰকাৰী বিৰোধক গ্ৰহীত কৰাৰ

পিছত ইয়াক বাস্তৱপতিৰ অন্মতিৰ কাৰণে প্ৰেৰণ
কৰা হয়। বাস্তৱপতিৰ অন্মতিৰ পিছত ই
আইনত পৰিণত হয় আৰু নিৰ্দিষ্ট অস্থায়
পৰা কাৰ্যকৰী হয়।

এইদৰে প্ৰেৰণীয় অংকিৰণে ১০৭ নং অনুচ্ছে
দৰ পৰা ১২২ নং অনুচ্ছেদলৈ বিধায়িনী
প্ৰক্ৰিয়াৰ আনুষ্ঠানিক ব্যৱস্থা, বিধায়কৰ প্ৰকাৰ
ইত্যাদি বিষয়বস্তুৰোৰ আলোচনা কৰিছে।
অংকিৰণত বিধায়িনী ব্যৱস্থাৰ দীৰ্ঘলীয়া
আলোচনা কৰাৰ উদ্দেশ্য হৈছে যিবি বিধান
সম্বন্ধত ক্ষতি সাধনা তৈয়াৰ কৰা। এই ক্ষেত্ৰত
আমাৰ অংকিৰণ নিৰ্মাতাসকল অক্ষল হৈছে
বুলি ক'ব পাৰি।

প্ৰসঙ্গ পুথি :- উক্ত কথাখিনি ৫° হুবুৰ।

আবুছ , বায় সুদৰ্শন , ৭° দত্ত দেৱদ্বিঃ এই
 কেইগৰাকী লেখকৰ “অন্তৰ্জাতীয় কাৰ্য - প্ৰণালী
 আৰু অনুশীলন” নামৰ কিতাপখনৰ
 পৰা লিখা হৈছে ।

পৃষ্ঠা নং :- 22-30; 41-46

(২)

(৩)

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1

প্রকল্প (PROJECT)

SEC COURSE

শিৰোনাম : চৌবতবর্ষে বিবেক ইহিতাবধন
প্রদীপা ।

জন্ম কর্তাব্দ : নিপিকা-বর্ষ

শ্রেণী : চুতীয় মাধ্যমিক

বোল নং : 163

2.

Certificate

This is to certify that

Miss Lipika Barman has worked under my Guidance and Supervision for her project work on SEC course, which is being submitted to the Dept of Political Science.

Date : 28/05/2023

Place : Nagasbema



(Dr. Arpani Kalita)
Asst. Professor

বিষয় সূচী - (content) পৃষ্ঠা নং -

বিবেক গৃহীতকৰণ অক্ৰিয়া —

পাতনি : 1

বিবেক গ্রন্থৰ পদ্ধতি : 5

সংসদীয় কাম আনালী : 12

সামৰণি : 45

উত্তৰঃ

① পাৰ্ভাণ :-

সংবিধান হৈছে কোনো এখন
দেশৰ শাসন ব্যৱস্থাৰ বৈধ দলিল আৰু
চুড়ান্ত আইন । ইয়াৰ জৰিয়তে দেশৰ শাসন
ব্যৱস্থা, বিধানমণ্ডলী ব্যৱস্থা আৰু ন্যায় ব্যৱস্থা,
আদিৰ লগত জড়িত বিষয়বোৰ সুসংগঠন পৰিচা-
লনা কৰা হয় । সংবিধানক ভিত্তি কৰি গতি
হুট কোনো এখন দেশৰ শাসন ব্যৱস্থা আৰু
জাতীয় পৰিচয় । দেশৰ স্বত্বাধীনতা, স্বাধীনতা, মানৱত্ব
চৰিত্ৰ, পৰম্পৰা, স্বাধীনতা, স্বাধীনতা, স্বাধীনতা
ইত্যাদি সকলো ক্ষেত্ৰতে সংবিধানৰ প্ৰত্যক্ষ
অথবা প্ৰতিপক্ষ প্ৰভাৱ পৰা দেখিবলৈ পোৱা যায় ।
সেয়েহে সংবিধানক এখন দেশৰ শাসন ব্যৱস্থা
আত্মা বুলি কোৱা হয় । ভাৰতীয় সংবিধানৰ
আটাইতকৈ গুৰুত্বপূৰ্ণ বস্তুটোৱেই আইন
প্ৰণয়ন কৰা । সংবিধানে ভাৰতীয় আইন
প্ৰণয়নৰ দায়িত্ব কেন্দ্ৰীয় আইন সভা আৰু
ৰাজ্যিক আইন সভাৰ মাজত ভাগ কৰিছে ।
কেন্দ্ৰীয় সংসদ কেন্দ্ৰীয় জালিকা হৈছে
বিষয় সূচী আৰু প্ৰমোদনৰ্থে ৰাজ্যিক
জালিকা হৈছে বিষয়সূচীৰ ওপৰতো আইন

প্ৰণয়ন কৰিব পাৰে।

২) বিল সন্মতিক, পৰিচয় :-

আইন প্ৰণয়নৰ উদ্দেশ্যে সংসদে যিহে
প্ৰস্তাৱ দাঙি ধৰা হয়। সেই প্ৰত্যেকটি
প্ৰস্তাৱকে বিল আকাৰে উপস্থাপন কৰা হয়
সংসদে উপস্থাপিত আইনৰ প্ৰচাৰ বা
প্ৰস্তাৱকে বিল বুলি কোৱা হয়।

বাংলাদেশৰ সংবিধানৰ ৮০ নং অনুচ্ছেদ
ত বিল সন্মতিকৰে উল্লেখ আছে।

সংসদৰ অনুমোদনৰ কাৰণে আইনৰ
মি প্ৰস্তাৱ উপস্থাপন কৰা হয় তাক
বিবেচক বোলে। গতিকে বিবেচক হলে
সংসদে প্ৰস্তাৱিত আইন এখনৰ প্ৰচাৰ
অৰ্থাৎ আইন পাণ্ডুলিপিৰ বিবেচক বা
বিল বোলা হয়। বিল এখনক
সংবিধানিক প্ৰক্ৰিয়া অনুসৰি স্থায়ী হোৱাৰ
পিছতহে আইনত পৰিণত হয়। এখন
বিবেচক এটা নিৰ্দিষ্ট কালোত্তৰে
আৱণ্ট হয়। বিবেচক এখনত
প্ৰস্তাৱনা আৰু কাৰ্যকৰণৰ সূত্ৰ থাকে।
তদুপৰি বিবেচক এখনৰ প্ৰয়োগ মোগ্যতা
পৰিমাণ দিয়া হয় আৰু কিছু ক্ষেত্ৰত
ই আইনত পৰিণত হোৱাৰ আৱশ্যক

নিৰ্দিষ্টকৈ দিয়া হয়।

বিল বা বিধেমক নিৰ্বাচিত প্ৰতিমাত্ম
সাদেশে সংসদে অনুমোদন জনালে আৰু বাস্তৱতামে
মতে তাত অনুমতি প্ৰদান কৰে তেন্তে ই আইনে
ক পালিত হয়। তেঁৱেৰে মতে কিছুমান বৈধানিক প্ৰকাৰ
মিলিলাক বিল অথবা বিধেমক ৰূপত সংসদৰ
অনুমতি দিয়া পৰা কৰা হয়।

বিধেমকৰ প্ৰকাৰ :

স্বতন্ত্ৰ কেন্দ্ৰীয় সংসদ হৈছে আইন প্ৰণয়নৰ

অৰ্হোক্ত সংস্থা (highest law making organ) সংসদৰ
টো সংসদ আৰু নিম্নসদন সমাজেৰে স্বাধীনতা আৰু
লোক সত্ৰৰ সুৰাৰ কাৰ্য হৈছে দেশৰ বাবে অস্বাভাৱিক
আইন ৰচনা কৰা, পুৰণি আইন সংশোধন বা পৰিৱৰ্তন
কৰি দেশৰ মানস চৰিত্ৰা প্ৰাণলিত কৰা। কেন্দ্ৰীয়
সংসদে প্ৰণয়ন কৰা বিধেমক বাস্তৱত প্ৰাণলিত
প্ৰয়োজন হয়। অৰ্থাৎ বাস্তৱ প্ৰাণলিত কিছুমান
বিধেমক আইনত পালিত হয়।

সাধাৰণতে সংসদে উল্লেখ কৰা
বিধেমক দুই ভাগত বিভক্ত কৰা হয়।

সেই দুটা হ'ল—

১) চৰকাৰী বিধেমক আৰু

২) বেচৰকাৰী বিধেমক।

সেইদৰে চৰকাৰী বিধেমক আৰু সাধাৰণ বিধেমক

কোনো বিষয়ত চৰকাৰৰ দৃষ্টি আকৰ্ষণ কৰা।

৭) এই বিবেকক ব্যৱসায় বা নোকসম্পত্তি উন্নয়ন
কৰিব পাৰি।

৮) সাধাৰণতে সংসদৰ বিৰোধী পক্ষৰ সাদৰ্য্যই বেচৰকাৰী
বিবেকক উন্নয়ন কৰা দেখা যায়।

৯) সামৰণি:

ওপৰত উল্লেখ কৰি গৈছে যে মতে দ্বিতীয়
সংবিধান মতে গঠন হোৱা প্ৰথমখন কেন্দ্ৰীয় চৰকাৰ
পৰা বৰ্তমানৰ কেন্দ্ৰীয় চৰকাৰৰ সময়লৈ প্ৰায় ৩০০খন
বেচৰকাৰী বিবেকক সংসদত উন্নয়ন হোৱা দেখা
যায়। সেই ৩০০ খন বিবেককৰ মাজৰ পৰা মাত্ৰ
২৪ খন বেচৰকাৰী বিবেকক সংসদৰ দ্বাৰা গৃহীত
হৈ গৈছে। মৰ্যদা লাভ কৰিছে। সংসদৰ এই
তথ্যই দেখুৱায় যে সংসদত মাত্ৰ ৪ শতাংশ
বেচৰকাৰী বিবেকক গৃহীত হোৱাৰ বিপৰীতে
৯৬ শতাংশ বেচৰকাৰী বিবেকক নাকচ হৈছে বা
সেইবোৰৰ ম্যাদ উকলি গৈছে।

মিমেই নহওক কিয়, অন্যান্য সাধাৰণ বিবেক
বা চৰকাৰী বিবেককৰ দৰে একেই প্ৰক্ৰিয়া
আৰু নীতি নিয়মৰ মাজেদি সংসদৰ উচ্চ
সদনে বেচৰকাৰী বিবেকক গৃহীত কৰাৰ
পিছত ইয়াক স্বাক্ষৰপত্ৰৰ অনুমতিৰ কাৰণে
প্ৰেৰণ কৰা হয়। স্বাক্ষৰপত্ৰৰ অনুমতিৰ -

পিছত ই আইন পৰিণত হয় আৰু
নিৰ্দিষ্ট সময়ৰ পৰা কাৰ্যকৰী হয়।

প্ৰশংসা পুস্তি :

৫° ফখীল কলিতা

দিবাকৰ দাস

পৃষ্ঠা →

৫° আব্দুছ ছব্ব

সুদৰ্শন বাহ

৫° দেৱজিৎ দত্ত

পৃষ্ঠা

২০২০: ১০০০দীয়

প্ৰতিয়া আৰু কাৰ্য পদ্ধতি
সমূহ।

২৬ - ৩৩

২০২০: ১০০০দীয় কাৰ্য প্ৰণালী

আৰু অনুশীলন।

ইউনিয়ন বুক পাব্লিকেশ্যন।

২৭, ৩০