BIMALA PRASHAD CHALIHA COLLEGE

~~PRACTICAL NOTE BOOK~~
~~DEPARTMENT OF MATHEMATICS~~

Project on
Gamma Function

DEPT. OF MATHEMATICS * B.P. CHALIHA COLLEGE * Estd.1974 NAGARBERA

Submitted by

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23/05/2023
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23/05/2023.

CERTIFICATE

This is certified that Arjufa Begum, a student of B.Sc 6th semester Mathematics (Honours) has carried out this project report " GUMMA FUNCTION by herself. She has fulfilled the requirements for the preparation of the project report for 4th paper of B.sc 6th semester examination.

She worked very hard to prepare this project. I wish him very success in her life.

23/5/23

Signature of Head of the Department

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Date: 23/5/23

Place: Nagarbera

ACKNOWLEDGEMENT

This Project report entitled" GUMMA FUNCTION" is intended for drawing attention of mathematical application in the field of

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Signature

Evolution Comittee:

1..........................Chairman / External — 23/5/23

2..........................H.O.D — 23/5/23

3..........................Supervisor — 23/05/23

CERTIFICATE OF ORIGINALTY

This certified that **Arjufa Begum**, a student of B.Sc 6th semester Mathematics (Hon) has carried out is report"GUMMA FUNCTION" under my supervision. She has fulfilled all requirements for preparation of the project report for 4th paper of B.Sc 6th semester examination.

She worked very hard prepare this project. I wish him very success in his life.

Signature of supervison

Date: 23/05/23

Place: Nagarbera

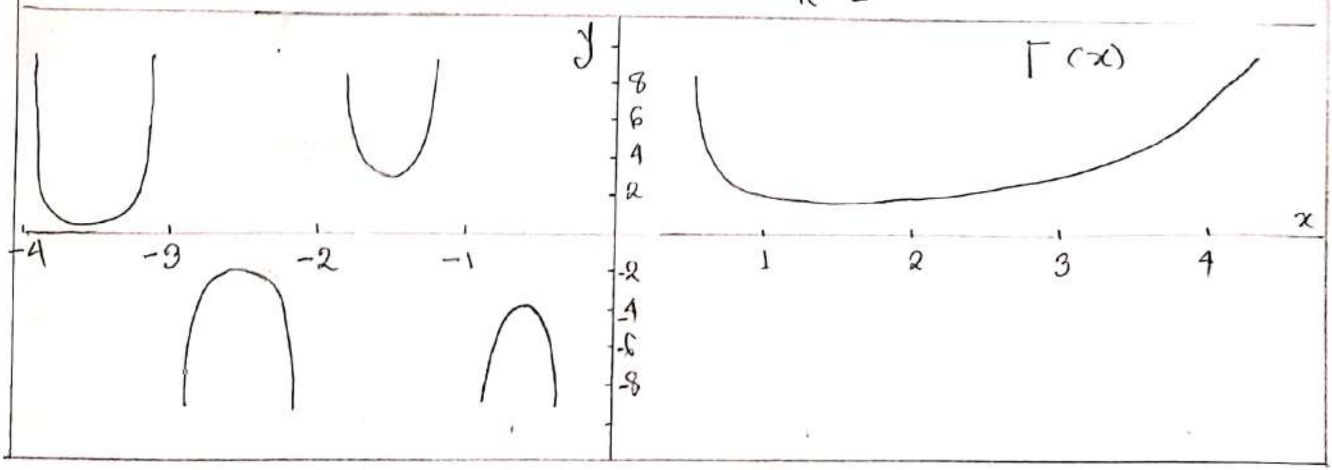
The Gamma function $\Gamma(x)$, is defined as

$$\Gamma(x) \equiv \int_0^{\infty} t^{x-1} e^{-t} \, dt,$$

Where $x \neq \ldots -4, -3, -2, -1, 0$.

Gamma function common rules and facts

- $\Gamma(x+1) \equiv x\Gamma(x)$, $x \in \mathbb{R}$
- $\Gamma(n+1) \equiv n!$ $n \in \mathbb{N}$ or $\Gamma(n) \equiv (n-1)!$, $n \in \mathbb{N}$
- $\Gamma(1) = 1$, $\Gamma(2) = 1$, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
- $\Gamma(0) = \pm\infty$, $\Gamma(-1) = \pm\infty$, $\Gamma(-2) = \pm\infty$, $\Gamma(-3) = \pm\infty$, etc
- $\Gamma(z)\Gamma(1-z) \equiv \dfrac{\pi}{\sin \pi z}$, $z \in \mathbb{C}$
- $\Gamma'(x) = \Gamma(x) \left[-\gamma + \frac{1}{x} + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{x+k} \right) \right]$



Question-1

Evaluate each of the following expressions, leaving the final answer in exact simplified form.

a) $\frac{3\Gamma(6)}{\Gamma(4)}$

b) $\Gamma\left(\frac{3}{2}\right)$

c) $\int_0^\alpha x^7 e^{-x} \, dx$.

V. 60, $\frac{1}{2}\sqrt{\pi}$, $7! = 5040$

a) $\frac{3\Gamma(6)}{\Gamma(4)} = \frac{3\times 5\,\Gamma(5)}{\Gamma(4)} = \frac{15\times 4\,\cancel{\Gamma(4)}}{\cancel{\Gamma(4)}} = 60$ //

b) $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$ //

c) $\int_0^\alpha x^7 e^{-x} \, dx \quad \int_0^\alpha x^{8-1} e^{-x} \, dx = \Gamma(8) = 7!$ //

Question - 2

Evaluate each of the following expressions, leaving the final answer in exact simplified form.

a) $\frac{60\Gamma(5)}{\Gamma(7)}$.

b) $\Gamma(\frac{5}{2})$.

c) $\int_0^\infty 2x^4 e^{1-x}\, dx$.

V, 2, $\frac{3}{4}\sqrt{\pi}$, 48e

a) $\frac{60\Gamma(5)}{\Gamma(7)} = \frac{60\Gamma(5)}{6\Gamma(6)} = \frac{60\cancel{\Gamma(5)}}{6\times 5\cancel{\Gamma(5)}} = 2$ //

b) $\Gamma(\frac{5}{2}) = \frac{3}{2}\Gamma(\frac{3}{2}) = \frac{3}{2}\times\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{3}{4}\sqrt{\pi}$ //

c) $\int_0^\infty 2x^4 e^{1-x}\, dx = \int_0^\infty 2x^{5-1} e^1 e^{-x}\, dx = 2e$

$\int_0^\infty x^{5-1} e^{-2}\, dx = 2e\,\Gamma(5) = 2e \times 4! = 48e$ //

Question 3.

Evaluate each of the following expression, leaving the final answer in exact Simplified form.

a) $\dfrac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})}$.

b) $\Gamma(-\frac{3}{2})$.

c) $\int_0^\infty x^3 e^{-4x}\, dx$.

V.-2, $\frac{4}{3}\sqrt{\pi}$, $\frac{3}{128}$

a) $\dfrac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})} = \dfrac{-\frac{1}{2}\Gamma(-\frac{1}{2})}{-\frac{1}{2}\Gamma(\frac{1}{2})} = \dfrac{\Gamma(-\frac{1}{2})}{-\frac{1}{2}\Gamma(-\frac{1}{2})} = \dfrac{1}{-\frac{1}{2}} = -2$

<u>OR</u> $\dfrac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})} = \dfrac{\cancel{\Gamma(-\frac{1}{2})}}{-\frac{1}{2}\cancel{\Gamma(-\frac{1}{2})}} = \dfrac{1}{-\frac{1}{2}} = -2$ //

b) $\Gamma(-\frac{3}{2}) \times (-\frac{3}{2}) = \Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = -\frac{3}{2}\Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = -\frac{2}{3} \times (-2) \times (-\frac{1}{2})\Gamma(-\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = \frac{4}{3}\Gamma(\frac{1}{2})$

$\Rightarrow \Gamma(-\frac{3}{2}) = \frac{4}{3}\sqrt{\pi}$ //

e) $\int_0^\alpha x^3 e^{-4x}\,dx = \ldots$ Substitution $\ldots = \int_0^\alpha \left(\frac{1}{4}t\right)^3 e^{-t}\frac{dt}{4}$

$= \frac{1}{256}\int_0^\alpha t^3 e^{-t}\,dt = \frac{1}{256}\int_0^\alpha t^{4-1} e^{-t}\,dt$

$= \frac{1}{256}\Gamma(4) = \frac{1}{256}\times 3! = \frac{6}{256}$

$= \frac{3}{128}$

$t = 4x$

$\frac{dt}{dx} = 4$

$dx = \frac{dt}{4}$

Question - 4

By using techniques involving the Gamma function, find the value of

$\int_0^\alpha x^3 e^{-\frac{1}{2}x^2}\,dx.$

V, IMB, 2

This can be Turned into A Gamma function by Substitution

$$\int_0^\alpha x^3 e^{-\frac{1}{2}x^2}\,dx = \int_0^\alpha x^3 e^{-u}\left(\frac{du}{x}\right)$$

$$= \int_0^\alpha x^2 e^{-u}\,du = \int_0^\alpha 2u e^{-u}\,du$$

$$\Gamma(\alpha) = \int_0^\alpha x^{\alpha-1} e^{-x}\,dx$$

$$= 2\int_0^\alpha u^{2-1} e^{-u}\,du = 2\,\Gamma(2)$$

$$= 2 \times 1! = 2 \;//$$

$u = \frac{1}{2}x^2$

$du = x\,dx$

$dx = \frac{du}{x}$

$2u = x^2$

$x = \sqrt{2u}$

Question - 5

By using techniques involving the Gamma function, find the value of

$$\int_0^\alpha \sqrt{x}\, e^{-\sqrt{x}}\,dx.$$

V, 4

$\int_0^\alpha \sqrt{x} e^{-\sqrt{x}} dx = \ldots$ By Substitution ....

$= \int_0^\alpha t e^{-t} (2t\, dt)$

$= 2 \int_0^\alpha t^2 e^{-t} dt = 2 \int t^{3-1} e^{-t} dt$

$= 2\Gamma(3) = 2 \times 2! = 4$ //

$t = \sqrt{x}$

$t^2 = x$

$2t\, dt = dx$

Question - 6

By using techniques involving the Gamma function, find the value of

$$\int_1^\alpha \frac{(\ln x)^3}{x^2} dx.$$

V, 6

$$\int_1^{\alpha} \frac{(\ln x)^3}{x^2}\,dx = \ldots \text{ by Substitution } \ldots = \int_0^{\alpha} \frac{u^3}{x^2}(x\,du)$$

$$= \int_0^{\alpha} \frac{u^3}{x}\,du = \int_0^{\alpha} \frac{u^3}{e^u}\,du = \int_0^{\alpha} u^3 e^{-u}\,du$$

$$= \int_0^{\alpha} u^{4-1} e^{-u}\,du = \Gamma(4) = 3! = 6 \,//$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$dx = x\,du$

$x = e^u$

$x = 1 \mapsto u = 0$

$x = \alpha \mapsto u = \alpha$

Question - 7

By using techniques involving the Gamma fuction. find the exact value of

$$\int_0^{\alpha} 2\sqrt{x}\, e^{-x^2}\,dx.$$

Give the answer in the form $\Gamma(K)$, where K is a rational constant.

$$V, \quad \Gamma\left(\frac{3}{4}\right)$$

$$\int_0^\alpha 2\sqrt{x}\, e^{-x^2}\, dx = \dots \text{By Substitution} \dots = \int_0^\alpha dt\, t^{\frac{1}{4}}\, e^{-t}\left(\frac{dt}{2x}\right)$$

$$= \int_0^\alpha t^{\frac{1}{4}}\, e^{-t}\frac{dt}{x} = \int_0^\alpha t^{\frac{1}{4}}\, e^{-t}\, \frac{dt}{t^{1/2}}$$

$$= \int_0^\alpha t^{-\frac{1}{4}}\, e^{-t}\, dt = \int_0^\alpha t^{\frac{3}{4}-1}\, e^{-t}\, dt$$

$$= \Gamma\left(\frac{3}{4}\right) \quad //$$

$t = x^2$

$t^{1/2} = x$

$t^{\frac{1}{4}} = x^{\frac{1}{2}}$

$dt = 2x\, dx$

Question - 8

By using techniques involving the Gamma function, find the exact value of

$$\int_0^\alpha x^6 e^{-4x^2} dx$$

Give the answer in the form $K\sqrt{\pi}$, where K is a rational constant.

$$\checkmark,\ \frac{15\sqrt{\pi}}{2048}$$

$$\int_0^{\alpha} x^6 e^{-4x^2} dx = \ldots \text{By Substitution} \ldots = \int_0^{\alpha} x^6 e^{-t} \frac{dt}{8x}$$

$$= \frac{1}{8} \int_0^{\alpha} x^5 e^{-t} dt = \frac{1}{8} \int_0^{\alpha} \left(\frac{t^{1/2}}{2}\right)^5 e^{-t} dt$$

$$= \frac{1}{256} \int_0^{\alpha} t^{5/2} e^{-t} dt = \frac{1}{256} \int_0^{\alpha} t^{\frac{7}{2}-1} e^{-t} dt$$

$$= \frac{1}{256} \Gamma\left(\frac{7}{2}\right) = \frac{1}{256} \times \frac{5}{2} \Gamma\left(\frac{5}{2}\right)$$

$$= \frac{1}{256} \times \frac{5}{2} \times \frac{3}{2} \Gamma\left(\frac{3}{2}\right) = \frac{1}{256} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{15}{2048} \Gamma\left(\frac{1}{2}\right) = \frac{15\sqrt{\pi}}{2048}$$

$t = 4x^2$

$dt = 8x\, dx$

$dx = \frac{dt}{8x}$

$x^2 = \frac{t}{4}$

$x = \frac{t^{1/2}}{2}$

Question - 9

By using techniques involving the Gamma function, find the exact value of

$$\int_0^1 \left[\ln\left(\frac{1}{x}\right)\right]^{a-1} dx$$

Where $a \neq 1, 0, -1, -2, -3, \ldots$

Give the answer in terms of a Gamma function.

V. $\quad , \Gamma(a)$

• Starting with a Substitution

$\Rightarrow t = \ln\left(\frac{1}{x}\right)$

$\Rightarrow t = -\ln x$

$\Rightarrow -t = \ln x$

$\Rightarrow x = e^{-t}$

$\Rightarrow dx = -e^{-t}\,dt$

Limits

$x = 0 \longrightarrow t = \infty$

$x = 1 \longrightarrow t = 0$

• the integral transforms to

$$\int_0^1 \left[\ln\left(\frac{1}{x}\right)\right]^{a-1} dx = \int_\infty^0 t^{a-1}(-e^{-t})\,dt$$

$$= \int_0^\infty t^{a-1} e^{-t}\,dt$$

which is the exact Definition of $\Gamma(a)$ //

Question 10

By using techniques involving the Gamma function.

Show that

$$\int_0^1 [\ln x]^n\,dx = (-1)^n n!\,,\quad n \in \mathbb{N}.$$

Proof:

$$\int_0^1 (\ln x)^n dx = \text{By Substitution} = \int_{\alpha}^{0} u^n e^u du$$

By another Substitution ... $= \int_{\alpha}^{0} (-t)^n e^{-t} (-dt)$

$-t = u$
$-dt = du$
$-\alpha \to \alpha$
$0 \to 0$

$$= \int_0^{\alpha} (-t)^n e^{-t} dt$$

$$= (-1)^n \int_0^{\alpha} t^u e^{-t} dt$$

$$= (-1)^n \Gamma(n+1)$$

$$= (-1)^n n!$$

$u = \ln x$
$du = \frac{1}{2} dx$
$dx = x\, du$
$dx = e^u du$

$x = 0,\ u = -\alpha$
$x = 1;\ u = 0$

Qustion - 11

By using techniques involving the Gamma function. Show that

$$\int_0^{\alpha} \frac{e^{-k\sigma^2}}{\sigma^6} d\sigma = \frac{3\sqrt{\pi}}{8k^{\frac{5}{2}}},\ k \neq 0$$

proof :-

$$\int_0^{\alpha} \frac{e^{\frac{-K}{\sigma^2}}}{\sigma^6}\, d\sigma = \ldots \text{By substitution}$$

• let $u = \frac{K}{\sigma^2}$

• limits $\sigma = 0 \to u = \alpha$
$\sigma = \alpha \to u = 0$

$$\sigma^2 = \frac{K}{u}$$

$$\sigma = \sqrt{K}\, u^{-\frac{1}{2}}$$

$$d\sigma = -\frac{1}{2}\sqrt{K}\, u^{-\frac{3}{2}}\, du$$

$$\ldots = \int_{\alpha}^{0} \frac{e^{-u}}{\frac{K^3}{u^3}} \left(-\frac{1}{2}\sqrt{K} u^{-\frac{3}{2}}\, du\right) = \int_0^{\alpha} \frac{u^3 e^{-u}}{K^3} \times \frac{\sqrt{K}}{2} u^{-\frac{3}{2}}\, du$$

$$= \frac{1}{2K^{5/2}} \int_0^{\alpha} e^{-u} u^{3/2}\, du = \frac{1}{2K^{5/2}} \Gamma\left(\frac{5}{2}\right) = \frac{1}{2K^{5/2}} \times \frac{3}{2} \times \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{3}{8K^{5/2}} \Gamma\left(\frac{1}{2}\right) = \frac{3\sqrt{\pi}}{8K^{5/2}} \;//$$

Question - 12

(a) Show clearly that

$$\Gamma(n+1) \equiv n\Gamma(n),\; n \neq 0 \;.\; n \in -\mathbb{N}.$$

b) Hence show further that

$$\Gamma(n+1) = n!\ , \quad n \in \mathbb{N}.$$

Proof:-

a) $\Gamma(n+1) = \int_0^\alpha t^{(n+1)-1} e^{-t} \, dt = \int_0^\alpha t^x e^{-t} \, dt = \ldots$

integration by parts

t^x	$x t^{x-1}$
$-e^{-t}$	e^{-t}

$$\ldots = \left[-t^x e^{-t}\right]_0^\alpha - \int_0^\alpha -x\, t^{x-1} e^{-t} \, dt = x \int_0^\alpha t^{x-1} e^{-t} dt$$

$$= x\,\Gamma(x) \;//$$

b) $\Gamma(n+1) = n\Gamma(n) = n(n-1)\Gamma(n-1) = n(n-1)(n-2)\Gamma(n-2) = \ldots$

$$= \ldots n(n-1)(n-2) \ldots 4 \times 3 \times 2 \times 1 \times \Gamma(1) = n!\,\Gamma(1)$$

$$= n! \cdot \int_0^\alpha t^{1-1} e^{-t} \, dt = n! \int_0^\alpha e^{-t} \, dt$$

$$= n! \left[-e^{-t}\right]_0^\alpha = n! \left[e^{-t}\right]_\alpha^0 = n! \left[1-0\right] = n! \;//$$

Question. 13

By using techniques involving the Gamma function, show that

$$\int_0^\infty x^m e^{-ax^n} dx = \frac{\Gamma\left(\frac{m+1}{n}\right)}{na^{\frac{m+1}{n}}}.$$

Proof:- $\int_0^\infty x^m e^{-ax^n} dx = \ldots$ By Substitution

$$= \int_0^\infty \frac{t^{\frac{m}{n}}}{a^{\frac{m}{n}+1}} \int_0^\infty t^{\frac{m}{n}} \left[\left(\frac{t}{u}\right)^{\frac{1}{u}}\right]^{1-n} e^{-t} dt = \frac{1}{na^{\frac{m+u}{u}}} \int_0^\infty t^{\frac{m}{u}} \times \frac{t^{\frac{m}{u}}}{a^{\frac{1-u}{u}}} e^{-t} dt$$

$$= \int_0^\infty \frac{t^{\frac{m}{u}}}{a^{\frac{m}{n}}} e^{-t} \frac{dt}{nax^{u-1}} = \int_0^\infty \frac{1}{a^{\frac{m}{n}} \times na} t^{\frac{m}{u}} x^{1-n-t} dt$$

$$= \frac{1}{na^{\frac{m}{u}+1}} \int_0^\infty \left[\left(\frac{t}{u}\right)^{\frac{1}{u}}\right]^{1-u} e^{t} dt = \frac{1}{na^{\frac{m+u}{u}}} \int_0^\infty t^{\frac{m}{u}} \times \frac{t^{\frac{m}{u}}}{a^{\frac{1-u}{u}}} \times \frac{t^{\frac{m}{u}}}{a^{\frac{1-u}{u}}} e^{-t} dt$$

$$= \frac{1}{na^{\frac{m+u}{n}} a^{\frac{1-u}{u}}} \int_0^\infty t^{\frac{m}{u}} \times t^{\frac{1-u}{u}} e^{-t} dt = \frac{1}{na^{\frac{m+1}{u}}} \int_0^\infty t^{\frac{m-n+1}{n}} e^{-t} dt$$

$$= \frac{1}{na^{\frac{m+1}{u}}} \int_0^\infty t^{\frac{m+1}{n}} e^{-t} dt = \frac{1}{na^{\frac{m+1}{n}}} \Gamma\left(\frac{m+1}{n}\right) //$$

$t = ax^n$

$dt = nax^{n-1}\,dx$

$dx = \frac{dt}{nax^{n-1}}$

$x = 0 \to t = 0$

$x = \alpha \to t = \infty$

$x^n = \frac{t}{a}$

$x = \left(\frac{t}{a}\right)^{\frac{1}{n}}$

$x^m = \left(\frac{t}{a}\right)^{\frac{m}{n}}$

Question - 14

By using techniques involving the Gamma function, Show that

$$\int_{-\alpha}^{k} \frac{e^{ax}}{(k-x)^n}\,dx = e^{ak} a^{n-1} \Gamma(1-n), \quad 0 < n < 1.$$

proof:- $\int_{\alpha}^{k} \frac{e^{ax}}{(k-x)^n}\,dx = \ldots$ By Substitution ...

$$= \int_{\alpha}^{0} \frac{e^{a(k-u)}}{u^n}(-dx)$$

$$= \int_0^{\alpha} \frac{e^{ak} e^{-u}}{u^n}\,du \quad \int_0^{\alpha} u^{-n} e^{-an}\,du$$

= ... Another Substitution

$$= e^{ak} \int_0^{\alpha} \left(\frac{t}{a}\right)^{-n} e^{-t} \frac{dt}{a} = e^{ak} \int_0^{\alpha} \frac{t^{-n} \times e^{-t}}{a^{-n} \times a}\,dt$$

$$= e^{ak} \times \frac{1}{a^{1-n}} \int_0^{\alpha} t^{-n} e^{-t}\,dt = e^{ak} a^{n-1} \Gamma(1-n) \,//$$

$$u = k - x$$
$$du = -dx$$

- - - - - - - -

$$x = -\alpha,\ u = \alpha$$
$$x = k,\ u = 0$$
$$x = k - u$$

$$t = au$$
$$dt = a\,du$$

Question - 15

The Gamma function is defined as

$$\Gamma(x) = \int_0^\alpha e^{-t} t^{x-1} dt,\ x \neq 0, -1, -2, -3 ..$$

i) Show that for $x > 0$

$$\Gamma(x+1) = x\Gamma(x)$$

ii) Express the integral

$$I(s) = \int_0^\alpha e^{-p^s} dp,\quad s > 0.$$

in terms of Gamma function.

iii) Deduce that

$$\lim_{s \to \alpha} [I(s)] = 1$$

$$I(s) = \frac{1}{s}\Gamma\left(\frac{1}{s}\right)$$

I) By definition $\Gamma(x) = \int_0^\alpha e^{-t} t^{x-1}\, dt$

This $\Gamma(x+1) = \int_0^\alpha e^{-t} t^{(x+1)-1}\, dt = \int_0^\alpha e^{-t} t^{x}\, dt$

integrating By parts w.R.T t

$$\begin{array}{c|c} t^x & x t^{x-1} \\ \hline -e^{-t} & e^{-t} \end{array}$$

$$\Gamma(x+1) = \left[-t^x \cancel{e^{-t}}\right]_0^\alpha + x\int_0^\alpha e^{-t} t^{x-1}\, dt$$

$$\Gamma(x+1) = x\int_0^\alpha e^{-t} t^{x-1}\, dt$$

$$\Gamma(x+1) = x\,\Gamma(x) \quad //$$

II) $I(s) = \int_0^\alpha e^{-p^s}\, dp$

$$I(s) = \int_0^\alpha e^{-t}\left(\frac{1}{s}p^{1-s}\right) dt$$

$$I(s) = \int_0^\alpha \frac{1}{s} e^{-t}\left(t^{\frac{1}{s}}\right)^{1-s} dt$$

$$I(s) = \int_0^\alpha \frac{1}{s} e^{-t} t^{\frac{1}{s}-1}\, dt$$

$$I(s) = \frac{1}{s}\Gamma\left(\frac{1}{s}\right) \quad //$$

III) $\lim_{s\to\alpha}[I(s)] = \lim_{s\to\alpha}\left[\frac{1}{s}\Gamma\left(\frac{1}{s}\right)\right] = \lim_{s\to\alpha}\left[\Gamma\left(\frac{1}{s}+1\right)\right]$

$= \Gamma(1) = 0! = 1$ //

Question 16

a) Write down the definition of the Gamma function $\Gamma(x)$, for $Re(x) > 0$.

b) Use the standard recurrence relation of the Gamma function to extent $\Gamma(x)$, for $Re(x) \leq 0$, and hence find the residues at the simple poles at $x = -n$, $n = 0, 1, 2, 3 \ldots$

$$\frac{(-1)^n}{n!}$$

a) $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$

b) Using the recurrence $\Gamma(x+1) = x\Gamma(x)$

$\Gamma(x) = \frac{\Gamma(x+1)}{x}$

Let $x = u-n$, where $n \in \mathbb{N}$ & $0 < u < 1$

$\Gamma(u-n) = \frac{\Gamma(u-n+1)}{u-n}$

$\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{\cancel{u-n+1}} \times \frac{\Gamma(u-n+2)}{u-n+1}$

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{\Gamma(u-n+3)}{u-n+2}$$

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{u-n+1} \times \frac{\Gamma(u-n+3)}{u-m+2} \cdots$$

$$\cancel{\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{u-n+1} \times \frac{\Gamma(u-n+2)}{u-n+1}}$$

$$\times \frac{1}{u-2} \; \frac{\Gamma(u)}{u-1}$$

$$\Gamma(u-n) = \frac{1}{u-n} \times \frac{1}{u-n+1} \times \frac{1}{u-n+2} \times \cdots \times$$

$$\frac{1}{u-2} \times \frac{1}{u-1} \times \frac{\Gamma(u+1)}{u}$$

$$\Gamma(u-n) = (-1)^n \frac{1}{(n-u)(n-u-1)(n-u-2)\cdots\times(2-u)(1-u)} \times \frac{\Gamma(u+1)}{u}$$

Hence residue

$$\lim_{u-0} \left[u\,\Gamma(u-n) \right]$$

$$= \lim_{u \to 0} \left[\frac{u(-1)^n \Gamma(u+1)}{(n-u)(u-u)(n-u-2)\cdots(2-u)(1-u)u} \right]$$

$$= \frac{(-1)^n \, \Gamma(1)}{n\,(n-1)(u-2)\cdots\times 3 \times 2 \times 1}$$

$$= \frac{(-1)^n}{n!}$$

Question - 17

$$I = 2\int_0^{\alpha} e^{-x^2}\, dx \text{ and } I = 2\int_0^{\alpha} e^{-y^2}\, dy.$$

By finding an expression for I, show that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

Proof :-

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^{\alpha} t^{\frac{1}{2}-1} e^{-t}\, dt = \int_0^{\alpha} t^{-1/2} e^{-t}\, dt = \ldots \text{Substitution}$$

$$= \ldots \int_0^{\alpha} u^{-1} e^{-u^2} (2u\,du) = 2\int_0^{\alpha} e^{-u^2}\, du$$

Now let $I = 2\int_0^{\alpha} e^{-x^2}\, dx$ and $I = 2\int_0^{\alpha} e^{-y^2}\, dy$

$$I^2 = \left[2\int_0^{\alpha} e^{-x^2} dx\right]\left[2\int_0^{\alpha} e^{-y^2} dy\right]$$

$$t^{1/2} = u$$
$$t = u^2$$
$$dt = 2u\,du$$

$$I^2 = 4\int_0^{\alpha}\int_0^{\alpha} e^{-x^2} e^{-y^2}\, dx\, dy$$

$$I^2 = 4\int_0^{\alpha}\int_0^{\alpha} e^{-(x^2+y^2)}\, dx\, dy$$

Now Switch into polar co. ordinates

$$I^2 = 4\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{\alpha} e^{-r^2}\, r\, dr\, d\theta$$

$$I^2 = 4\int_{\theta=0}^{\frac{\pi}{2}} \left[-\frac{1}{2} e^{-r^2}\right]_0^{\alpha} d\theta$$

$$I^2 = 2\int_{\theta=0}^{\frac{\pi}{2}} \left[e^{-r^2}\right]_{\alpha}^{0} d\theta$$

$$I^2 = 2\int_{\theta=0}^{\pi/2} 1 \, d\theta$$

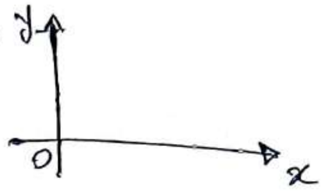
$$I^2 = 2 \times [\theta]_0^{\pi/2}$$

$$I^2 = \pi$$

$$\therefore \; I = \sqrt{\pi}$$

$$\Gamma(1/2) = \sqrt{\pi}$$

- $x^2 + y^2 = r^2$
- $dx\,dy = r\,dr\,d\theta$
-



$0 \leq x < \infty$
$0 \leq y < \infty$
Because

$0 \leq r < \infty$
$0 \leq \theta \leq \frac{\pi}{2}$

Question - 18

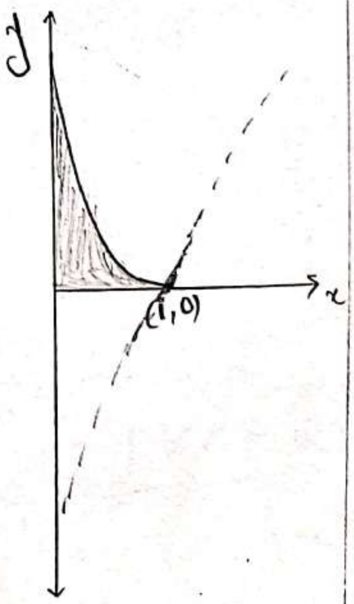
It is given that ~~above~~ the following integral converges

$$\int_0^1 [\ln x]^{1/2} \, dx .$$

Evaluate the above integral by a suitable substitution introducing Gamma functions.

You may assume that

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi} .$$



$\sqrt{\pi}$

$$\int_0^1 \left[\ln x\right]^{-1/2} dx = \cdots$$

As $y = \ln x$ is Negative in $[0,1]$

$$\int_0^1 [\ln x]^{-1/2}\, dx = \left[\int_0^1 (\ln x)^{-1/2}\right] dx$$

$$\ldots = \left|\int_\alpha^0 (\ln x)^{-1/2}\, dx\right|$$

By Substitution

$$\ldots = \left|\int_\alpha^0 (-u)^{-\frac{1}{2}} (-e^{-u}\, du)\right|$$

$\ln x = -u$
$u = \ln x$
$e^{-u} = x$
$dx = -e^{-u} du$
$x = 0 \quad u = +\alpha$
$x = 1 \quad u = 0$

$$= \left|\int_0^\alpha (-u)^{-1/2} e^{-u}\, du\right|$$

$$= \left|\int_0^\alpha (-1)^{-1/2} e^{-u}\, du\right|$$

$$= \left|(-1)^{-1/2} \int_0^\alpha u^{-1/2} e^{-u}\, du\right|$$

$$= \left|(e^{i\pi})^{-\frac{1}{2}} \int_0^\alpha e^{-u} u^{-1/2}\, du\right|$$

$$= \left|e^{\frac{-i\pi}{2}} \int_0^\alpha e^{-u} u^{-1/2-1}\, du\right|$$

$$= \left|e^{-i\frac{\pi}{2}} \int_0^\alpha e^{-u} u^{1/2-1}\, du\right|$$

$$= \left|e^{-i\frac{\pi}{2}}\right| \left|\Gamma(1/2)\right|$$

$$= \Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi} \quad //$$

$$\left\{ \left|e^{i\theta}\right| = 1 \ \forall \theta \in \mathbb{R} \right\}$$

Question - 19

It is given that the following integral Converges

$$\int_0^1 (x \ln x)^n \, dx, \; n \in \mathbb{N}.$$

Evaluate the above integral by a Substitution introducing Gamma functions.

$$\ldots, \quad \frac{(-1)^n n!}{(n+1)^{n+1}}$$

Start by a Standard Substitution

$u = -\ln x$ ← in order to Revers the low unit to $+\infty$

$$\frac{du}{dx} = -\frac{1}{x}$$

$dx = -x\,du$

$dx = -e^{-u}\,du$ ← Since $-u = \ln x$

$x = e^{-u}$

To the units transform

$x = 0 \longrightarrow u = \infty$

$x = 1 \longrightarrow u = 0$

Transforming the intergral

$$\int_0^1 (x\ln x)^n dx = \int_\infty^0 [e^{-u}(-u)]^n(-e^{-u}du) = \int_0^\infty e^{-nu}(-u)^n e^{-u} du$$

$$= (-1)^n \int_0^\infty e^{-nu} e^{-u} u^n du = (-1)^n \int_0^\infty e^{-(n+1)u} u^n du$$

~~linear~~ Simple linear Substitution to Turn

into ~~a~~ Gamma function

$$t = (n+1)u \iff u = \frac{t}{n+1}$$

$$\implies du = \frac{1}{n+1} dt$$

~~units~~

$$\cdots = (-1)^n \int_0^\infty e^{-t}\left(\frac{t}{n+1}\right)^n \left(\frac{1}{n+1} dt\right)$$

$$= (-1)^n \times \frac{1}{(n+1)^{n+1}} \int_0^\infty e^{-t} t^n dt$$

Using the Defination of the Gamma function

$$\cdots = \frac{(-1)^n}{(n+1)^{n+1}} \times \Gamma(n+1)$$

$$= \frac{(-1)^n}{(n+1)^{n+1}} \times n!$$

$$\therefore \int_0^1 (x\ln x)^n dx = \frac{(-1)^n n!}{(n+1)^{n+1}}$$

Question - 20

It is Given that the following integral Converges

$$\int_0^\infty e^{-\frac{1}{2}t} \ln t \, dt .$$

Evaluate the above integral by introducing a new parametric term in the integrand and carrying out a ~~St.~~ suitable differentiation under the integral sign.

you may assume that

$$\Gamma'(x) = \Gamma(x)\left[-\gamma + \frac{1}{x} + \sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{x+k}\right)\right]$$

$$I = \int_0^\infty e^{-\frac{1}{2}t} \ln t \, dt \qquad 2(-\gamma + \ln 2$$

- This has the signature of a Gamma function

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt$$

- introduce a parameter x, As in the Gamma function

$\Rightarrow$ let $J = \int_0^\infty t^{x-1} e^{-\frac{1}{2}t} \ln t \, dt$ $\boxed{\text{Where } I \text{ is } J \text{ with } x=1}$

- Let Observe that $\frac{d}{dx}(t^{x-1}) = t^{x-1} \times \ln t \times 1 = t^{x-1} \ln t$

• This we May write J as

$$\Rightarrow J = \int_0^{\alpha} \frac{J}{Jx} \left[t^{x-1} e^{-\frac{1}{2}t} \right] dt = \frac{J}{Jx}\left[\int_0^{\alpha} t^{x-1} e^{-\frac{1}{2}t} dt \right]$$

without is as most A Gamma function

$$\boxed{\begin{array}{l} \text{let } u = \frac{1}{2}t \iff t = 2u \\ du = \frac{1}{2}dt \\ dt = 2du \\ \text{~~Limits~~} \end{array}}$$

$$\Rightarrow J = \frac{J}{Jx}\left[\int_0^{\alpha} (2u)^{x-1} e^{-u} (2du) \right] = \frac{J}{Jx}\left[\int_0^{\alpha} 2^{x-1-u} e\, dt \right]$$

$$\Rightarrow J = \frac{J}{Jx}\left[2^{x} \int_0^{\alpha} e^{-u} u^{x-1} du \right] = \frac{J}{Jx}\left[2^{x} \Gamma(x) \right]$$

• Differentiating the proof

$$\Rightarrow J = 2^{x} \ln 2\, \Gamma(x) + 2^{x} \Gamma'(x)$$

$$\Rightarrow I = J(1) = 2 \ln 2\, \Gamma(1) + 2\Gamma'(1)$$

$$\Rightarrow J = 2 \ln 2 + 2\Gamma'(1) \qquad (\Gamma(1) = 0! = 1)$$

• Now $\Gamma'(x) = \Gamma(x)\left[-\gamma - \frac{1}{x} + \sum_{k=1}^{\alpha} \left[\frac{1}{k} - \frac{1}{x+k} \right.\right.$

$$\Rightarrow \Gamma'(1) = \Gamma(1)\left[-\gamma - 1 + \sum_{k=1}^{\alpha} \left[\frac{1}{k} - \frac{1}{k+1} \right] \right.$$

$$\Rightarrow \Gamma'(1) = \gamma - 1 + \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} \cdots \right]$$

$$\Rightarrow \Gamma'(1) = -\gamma - 1 \cancel{+} + 1$$

$$\Rightarrow \Gamma'(1) = -\gamma$$

$$\Rightarrow I = 2\ln 2 + 2(-\gamma)$$

$$\Rightarrow \int_0^\infty e^{-\frac{1}{2}t} \ln t \, dt = 2\left[-\gamma + \ln 2\right] \; //$$

Question - ~~1~~ 21

prove the validity of Legendre's duplication formular for the Gammar function, which States that

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{\Gamma(2n)\sqrt{\pi}}{2^{2n-1}\Gamma(n)}, \; n \in \mathbb{N}.$$

proof :-

Start by the ~~reverse~~ recovers property of this Gammar function

$$\Gamma(n + 1/2) = \left(n - \frac{1}{2}\right)\left(n - \frac{3}{2}\right)\left(n - \frac{5}{2}\right) \cdots \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\,\Gamma(1/2)$$

$$= \tfrac{1}{2}(2n-1) \times \tfrac{1}{2}(2n-3) \times \frac{1}{2}(2n-5) \cdots \left(\frac{1}{2} \times 7\right)$$
$$\left(\frac{1}{2} \times 5\right)\left(\tfrac{1}{2} \times 3\right)\left(\frac{1}{2} \times 1\right) \times \sqrt{\pi}$$

$$= \left(\frac{1}{2}\right)^n (2n-1)(2n-3)(2n-5) \cdots \cancel{\left(\frac{1}{2} \times 7\right)} \cancel{\left(\frac{1}{2} \times 5\right)}$$
$$\cancel{\left(\frac{1}{2} \times 3\right)\left(\frac{1}{2} \times 1\right) \times \sqrt{\pi}}$$
$$\cdots 7 \times 5 \times 3 \times 1 \times \sqrt{\pi}$$

$$= \frac{1}{2^n} \times \frac{(2n-1)(2n-2)(2n-3)(2n-4)\,2n-5) \cdots 7 \times 6 \times 5 \times 4 \times 3 \times 2}{(2n-2)(2n-4)(2n-6) \cdots 6 \times 4 \times 2} \times \sqrt{\pi}$$

$$= \frac{1}{2} \times \frac{(2n-1)!\,\sqrt{\pi}}{2^{n-1} \times (n-1)(n-2)(n-3) \cdots \times 3 \times 2 \times 1}$$

$$= \frac{1}{2^n} \times \frac{\Gamma(2n)\sqrt{\pi}}{2^{n-1}\,(n-1)!}$$

$$= \frac{\Gamma(2n)\sqrt{\pi}}{2^{n-1}\,(n-1)!}$$

//

Question:- 22

legendre's duplication formula for the Gamma function states that

$$\Gamma(m+\tfrac{1}{2}) = \frac{\Gamma(2m)\sqrt{\pi}}{2^{2m-1}\Gamma(m)}, \; m \in \mathbb{N}.$$

Show that

$$\Gamma(m+\tfrac{1}{2}) = \frac{(2m)!\sqrt{\pi}}{2^{2m}\, m!}, \; m \in \mathbb{N}.$$

Proof: (By legendre's duplication formula.

$$\Gamma(m+\tfrac{1}{2}) = \frac{\Gamma(2m)\sqrt{\pi}}{2^{2m-1}\Gamma(m)} = \frac{(2m-1)!\sqrt{\pi}}{2^{2m}\times \tfrac{1}{2}(m-1)!} = \frac{2(2m-1)!\sqrt{\pi}}{2^{2m}(m-1)!}$$

$$= \frac{2m(2m-1)!\sqrt{\pi}}{2^{2m}\, m(m-1)!} = \frac{(2m)!\sqrt{\pi}}{2^{2m}\, m!} \; //$$

P.T.O

Question 23.

$$I(\lambda, x) = \int_0^\infty e^{-\lambda} t^{x-1} \ln t \, dt.$$

ⓐ By carrying a suitable differentiable over the integral sign, show that.

$$I(\lambda, x) = \lambda^{-x}\left[\Gamma'(x) - \Gamma(x) \ln \lambda \right].$$

ⓑ Find simplified expressions for $I(\lambda, 1)$, $I(\lambda, 2)$ and $I(\lambda, 3)$.

You may assume that $\Gamma'(x) = \Gamma(x)\left[-\gamma + \frac{1}{x} + \sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{x+k}\right)\right]$.

$$I(\lambda, 1) = -\frac{1}{\lambda}\left[\gamma + \ln \lambda\right], \quad I(\lambda, 2) = \frac{1}{\lambda^2}\left[1 - \gamma - \ln \lambda\right],$$

$$I(\lambda, 3) = \frac{1}{\lambda^3}\left[3 - 2\gamma - 2\ln \lambda\right].$$

a) <u>Start by noting that</u> $\frac{d}{dx}(a^x) = a^x \ln a$.

$$\int_0^\infty e^{-\lambda t} t^{x-1} \ln t \, dt = \int_0^\infty \frac{\partial}{\partial x}\left[e^{-\lambda t} t^{x-1}\right] dt.$$

$$= \frac{\partial}{\partial x} \int_0^\infty e^{-\lambda t} t^{x-1} \, dt.$$

~~As This ALMOST~~ /

<u>As this Almost looks like A Gamma function we use A simple ~~unea.~~ Linear substitution.</u>

$$\boxed{\begin{aligned} u &= \lambda t \\ \frac{du}{dt} &= \lambda. \\ dt &= \frac{1}{\lambda} du. \end{aligned}}$$

$$= \frac{\partial}{\partial x} \int_0^\infty e^{-u} \left(\frac{u}{\lambda}\right)^{x-1} \left(\frac{1}{\lambda} du\right).$$

$$= \frac{\partial}{\partial x} \int_0^\infty e^{-u} u^{x-1} \frac{1}{\lambda^x} du.$$

$$= \frac{\partial}{\partial x} \left[\lambda^{-x} \int_0^\infty e^{-u} u^{x-1} du.\right.$$

$$= \frac{\partial}{\partial x}\left[\gamma^{-x}\,\Gamma(x)\right].$$

<u>Differentiating w.r.t. x, using product Rule:</u>

$$= \gamma^{-x}(-1)\ln\gamma\,\Gamma(x) + \gamma^{-x}\,\Gamma'(x)$$

$$= \gamma^{-x}\left[\Gamma'(x) - \Gamma(x)\ln\gamma\right] //$$

(b) Start by computing simplified expressed, in terms of γ for the Definition of Gamma for $x = 1, 2, 3 \ldots$

using $\left[\Gamma'(x) = \Gamma(x)\left[-\gamma - \frac{1}{x} + \sum_{K=1}^{\infty}\left(\frac{1}{K} - \frac{1}{x+K}\right)\right.\right.$.

- $\Gamma'(1) = \Gamma(1)\left[-\gamma - 1 + \sum_{K=1}^{\infty}\left(\frac{1}{K} - \frac{1}{K+1}\right)\right]$

$$= 0!\left[-\gamma - 1 + 1 - \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}} + \cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}} + \cancel{\tfrac{1}{4}} - \ldots\right]$$

$$= 1\left[-\gamma - 1 + 1\right] = -\gamma$$

- $\Gamma'(2) = \Gamma(2)\left[-\gamma - \frac{1}{2} + \sum_{K=1}^{\infty}\left(\frac{1}{K} - \frac{1}{K+2}\right)\right]$

$$= 1!\left[-\gamma - \tfrac{1}{2} + 1 + \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \ldots - \tfrac{1}{3} - \tfrac{1}{4} - \tfrac{1}{5} \ldots\right]$$

$$= 1\cdot\left[-\gamma - \cancel{\tfrac{1}{2}} + 1 + \cancel{\tfrac{1}{2}}\right] = 1 - \gamma$$

- $\Gamma'(3) = \Gamma(3)\left[-\gamma - \frac{1}{3} + \sum_{K=1}^{\infty} \frac{1}{K} - \frac{1}{K+3}\right]$

$$= 2!\left[-\gamma - \tfrac{1}{3} + 1 + \tfrac{1}{2} + \cancel{\tfrac{1}{4}} + \cancel{\tfrac{1}{5}} + \cancel{\tfrac{1}{6}} + \ldots - \cancel{\tfrac{1}{4}} - \cancel{\tfrac{1}{5}} - \cancel{\tfrac{1}{6}} - \ldots\right]$$

$$= 2\left[-\gamma - \tfrac{1}{3} + 1 + \tfrac{1}{2} + \tfrac{1}{3}\right] = 3 - 2\gamma.$$

Finaly we obtain:

$$I(\lambda, 1) = \lambda^{-1}\left[\Gamma'(1) - \Gamma(1)\ln\lambda\right] = \tfrac{1}{\lambda}\left[-\gamma - 0!\ln\lambda\right]$$
$$= -\tfrac{1}{\lambda}(\gamma + \ln\lambda).$$

$$I(\lambda, 2) = \lambda^{-2}\left[\Gamma'(2) - \Gamma(2)\ln\lambda\right] = \tfrac{1}{\lambda^2}\left[1 - \gamma - 1!\ln\lambda\right]$$
$$= \tfrac{1}{\lambda^2}(1 - \gamma - \ln\lambda)$$

$$I(\lambda, 3) = \lambda^{-3}\left[\Gamma'(3) - \Gamma(3)\ln\lambda\right] = \tfrac{1}{\lambda^3}\left[(3 - 2\gamma) - 2!\ln\lambda\right]$$
$$= \tfrac{1}{\lambda^3}\left[3 - 2\gamma - 2\ln\lambda\right]$$

Question 24

By considering standard power series expansions and using gamma functions, show

$$\int_0^1 x^x\,dx = \sum_{r=1}^{\infty}\left[\frac{(-1)^{r-1}}{r^r}\right]$$

Assume that integration and summation commute.

Proof:

Rewriting the integration

$$\int_0^1 x^x\,dx = \int_0^1 e^{\ln x^x}\,dx = \int_0^1 e^{x\ln x}\,dx$$

Rewrite as the exponential series of integrating ~~integration~~ integration / Summation.

$$= \int_0^1 \sum_{r=0}^{\infty} \frac{(x\ln x)^r}{r!}\, dx = \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^1 (x\ln x)^r\, dx \right.$$

Now using a substitution (Then minus, to the take the limits to 0 to ∞.

$$t = -\ln x$$
$$-t = \ln x$$
$$x = e^{-t}$$
$$dx = -e^{-t} dt.$$
$$x = 0 \longmapsto t = 0$$
$$x = 1 \longmapsto t = 0$$

$$= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_{\infty}^{0} [e^{-t}(-t)]^r (-e^{-t} dt) \right]$$

$$= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^{\infty} (-te^{-t})^r e^{-t} dt \right]$$

$$= \sum_{r=0}^{\infty} \left[\frac{1}{r!} \int_0^{\infty} (-1)^r e^{-tr} e^{-t}\, t^r\, dt \right]$$

Tioy or futhere. Ey employ another substitution.

$$u = t(r+1), \quad t = \frac{u}{r+1}$$
$$du = dt\,(r+1)$$
$$dt = \frac{du}{r+1}$$

$$= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} t^r e^{-t(r+1)}\, dt \right.$$

$$= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} \left(\frac{u}{r+1}\right)^r e^{-u} \left(\frac{du}{r+1}\right) \right]$$

$$= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \int_0^{\infty} \frac{u^r}{(r+1)^r}\, e^{-u} \frac{1}{r+1}\, du \right]$$

$$= \sum_{r=0}^{\infty} \left[\frac{(-1)^r}{r!} \times \frac{1}{(r+1)^{r+1}} \int_0^{\infty} u^r e^{-u}\, du \right]$$

Now by definition this is a gamma function

$$\Gamma(x) \equiv \int_0^{\infty} u^{x-1} e^{-u}\, dx \equiv (x-1)!$$

$$\Gamma(r+1) = \int_0^{\infty} u^r e^{-u}\, du = r!$$

Finaly we have,

$$\int_0^1 x^x\,dx = \sum_{r=0}^{\infty}\left[\frac{(-1)^r}{r!}\times\frac{1}{(r+1)^{r+1}}\int_0^{\infty} u^r e^{-u}\,du\right]$$

$$\int_0^1 x^x\,dx = \sum_{r=0}^{\infty}\left[\frac{(-1)^r}{r!}\times\frac{1}{(r+1)^{r+1}}\times r!\right]$$

$$\int_0^1 x^x\,dx = \sum_{r=0}^{\infty}\frac{(-1)^r}{(r+1)^{r+1}}$$

$$\cancel{\therefore \int_0^1 x^x\,dx = \sum_{r=1}^{\infty}\frac{(-1)^r}{(r+1)^{r+1}}}$$

$$\int_0^1 x^x\,dx = \sum_{r=1}^{\infty}\frac{(-1)^{r-1}}{r^r} \quad //$$

$$1 - 1/2^2 + 1/3^3 - 1/4^4 + 1/5^5 - 1/6^6 + 1/7^7 - \ldots\ldots$$

Question 25

By considering standard power series expansions and using Gamma functions. show

$$\int_0^1 \frac{1}{x^x}\,dx = 1 + 1/2^2 + 1/3^3 + \frac{1}{4^4} + \frac{1}{5^5} + \ldots\ldots$$

Assume that integration and summation commute.

Proced As follows:

Proof:

$$\int_0^1 \frac{1}{x^x}\,dx = \int_0^1 x^{-x}\,dx = \int_0^1 e^{\ln x^{-x}}\,dx = \int_0^1 e^{-x\ln x}\,dx.$$

Using the exporting series

$$\int_0^1 \sum_{r=0}^{\infty} \frac{(-x\ln x)^r}{r!}\,dx = \sum_{r=0}^{\infty}\left[\frac{1}{r!}\int_0^1 (-1)^r x^r (\ln x)^r\,dx\right]$$

$$= \sum_{r=0}^{\infty}\left[\frac{(-1)^r}{r!}\int_0^1 x^r (\ln x)^r\,dx\right]$$

Now using a substitution to create a gamma function (use a minus to get the "correct" limits).

$$\boxed{\begin{aligned} t &= -\ln x \\ -t &= \ln x \\ x &= e^{-t} \\ dx &= -e^{-t}\,dt \\ \hline x = 0 &\longmapsto t = \infty \\ x = 1 &\longmapsto t = 0 \end{aligned}}$$

$$= \sum_{r=0}^{\infty}\left[\frac{(-1)^r}{r!}\int_{\infty}^{0} e^{-tr}(-t)^r(-e^{-t})\,dt\right]$$

$$= \sum_{r=0}^{\infty}\left[\frac{(-r)^r}{r!}\int_0^{\infty} t^r(-1)^r e^{-tr}e^{-t}\,dt\right]$$

$$= \sum_{r=0}^{\infty}\left[\frac{(-1)^r(-1)^r}{r!}\int_0^{\infty} t^r e^{-t(r+1)}\,dt\right]$$

Another substitution to create a "full Gamma"

$$\boxed{\begin{aligned} u &= t(r+1) \\ du &= dt(r+1) \\ t &= \frac{u}{r+1} \\ dt &= \frac{du}{r+1} \end{aligned}}$$

$$= \sum_{r=0}^{\infty}\left[\frac{(-1)^{2r}}{r!}\int_0^{\infty}\left(\frac{u}{r+1}\right)^r e^{-u}\left(\frac{du}{r+1}\right)\right]$$

$$= \sum_{r=0}^{\infty}\left[\frac{1}{r!}\int_0^{\infty}\frac{u^r}{(r+1)^r}\,e^{-u}\,\frac{1}{r+1}\,du\right]$$

$$= \sum_{r=0}^{\infty}\left[\frac{1}{r!}\,\frac{1}{(r+1)^{r+1}}\int_0^{\infty} u^r e^{-u}\,du\right]$$

Now this a gamma function.

$$\Gamma(r) \equiv \int_0^{\infty} u^{r-1} e^{-u}\, du = (r-1)!$$

$$\Gamma(r+1) = \int_0^{\infty} u^{r} e^{-u}\, du = r!$$

Thus we have.

$$\int_0^1 \frac{1}{x^x}\, dx = \sum_{r=0}^{\infty} \left[\frac{1}{r!(r+1)^{r+1}} \Gamma(r+1) \right]$$

$$\int_0^1 \frac{1}{x^x}\, dx = \sum_{r=0}^{\infty} \left[\frac{1}{r!(r+1)^{r+1}} (r!) \right]$$

$$\int_0^1 \frac{1}{x^x}\, dx = \sum_{r=0}^{\infty} \frac{1}{(r+1)^{r+1}} = \sum_{r=1}^{\infty} \frac{1}{r^r}$$

$$\therefore \int_0^1 \frac{1}{x^x}\, dx = 1 + 1/2^2 + 1/3^3 + 1/4^4 + 1/5^5 + 1/6^6 + \ldots \quad //$$

Question 26

$$I = \int_1^{\infty} \frac{(\ln x)^3}{x^2(x-1)}\, dx$$

Show by the means of partial fractions and suitable substitution that

$$I = -6 + \int_0^{\infty} u^3 e^{-u} (1 - e^{-u})^{-1}\, du,$$

and hence show that $I = \frac{\pi^4}{15} - 6$

Assume without proof that.

$$\zeta(4) = \sum_{r=1}^{\infty}\left[\frac{1}{r^4}\right] = \frac{\pi^4}{90}$$

$$\int_1^{\infty} \frac{(\ln x)^3}{x^3(x-1)}\,dx = \frac{\pi^4}{15} - 6$$

<u>Proof:</u>

- Start by partial fraction

$$\frac{1}{x^2(x-1)} \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$$

$$1 \equiv A(x-1) + Bx(x-1) + Cx^2$$

$$1 \equiv (B+C)x^2 + (A-B)x - A$$

$$\therefore\ A=-1,\quad B=-1,\quad C=1$$

- Rewrite the Integral

$$\Rightarrow I = \int_0^{\infty}\left[\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}\right](\ln x)^3\,dx$$

- By substitution next

$$u = \ln x \qquad x=1 \longmapsto u=0$$

$$e^u = x \qquad x=\infty \longmapsto u=\infty$$

$$dx = e^u\,du$$

$$\Rightarrow I = \int_0^{\infty}\left[\frac{1}{e^u-1} - \frac{1}{e^u} - \frac{1}{e^{2u}}\right]u^3(e^u\,du)$$

$$\Rightarrow I = \int_0^{\infty}\left[\frac{e^u}{e^u-1} - 1 - \frac{1}{e^u}\right]u^3\,du$$

$$\Rightarrow I = \int_0^{\alpha} u^3 \left[\frac{e^u - e^u + 1}{e^u - 1}\right] du - \int_0^{\alpha} u^3 e^{-u} du$$

$$\cancel{\Rightarrow I = \int_0^{\alpha} u^3 \left(\frac{e^{-u}}{1-e^{-u}}\right) du}$$

$$\Rightarrow I = \int_0^{\alpha} \frac{u^3}{e^u - 1} du - \Gamma(4)$$

$$\Rightarrow I = \int_0^{\alpha} u^3 \left(\frac{e^{-u}}{1-e^{-u}}\right) du - 3!$$

$$\Rightarrow I = \int_0^{\alpha} u^3 e^{-u} (1 - e^{-u})^{-1} du - 6$$

$$\Rightarrow I = \int_0^{\alpha} u^3 e^{-u} \left[1 + e^{-u} + e^{-2u} + e^{-3u} + \cdots\right] du - 6$$

$$\boxed{\begin{gathered} \frac{1}{1-r} = 1 + r + r^2 + r^3 + \cdots \quad |r| < 1 \\ \text{Here, } |e^{-u}| = \left|\frac{1}{e^u}\right| = \frac{1}{x} \quad x > 1 \end{gathered}}$$

$$\Rightarrow I = \int_0^{\alpha} u^3 \left[e^{-u} + e^{-2u} + e^{-3u} + e^{-4u} + \cdots\right] du - 6$$

$$\Rightarrow I = \int_0^{\alpha} u^3 \left(\sum_{r=1}^{\alpha} e^{-ru}\right) du - 6$$

$$\Rightarrow I = \sum \int_0^{\alpha} u^3 e^{-ru} du - 6$$

- By part 3 times ; Differentiation w.r.t r under the integrate sign, or switch into Gamma By substitution

$$v = \gamma u$$
$$u = \frac{1}{\gamma} v$$
$$du = \frac{1}{\gamma} dv$$

$$\Rightarrow I = \sum_{r=1}^{\infty} \int_0^{\infty} \left(\frac{1}{\gamma} v\right)^3 e^{-v} \frac{1}{\gamma} dv - 6$$

$$\Rightarrow I = \sum_{r=1}^{\infty} \int_0^{\infty} \frac{1}{\gamma^4} v^3 e^{-v} dv - 6$$

$$\Rightarrow I = \sum_{r=1}^{\infty} \frac{1}{\gamma^4} \int_0^{\infty} v^3 e^{-v} dv - 6.$$

$$\Rightarrow I = \sum_{\gamma=1}^{\infty} \left[\frac{1}{\gamma^4} \Gamma(4)\right] - 6$$

$$\Rightarrow I = \left[\sum_{r=1}^{\infty} \frac{3!}{\gamma^4}\right] - 6$$

$$\Rightarrow I = 6 \left[\sum_{r=1}^{\infty} \frac{1}{\gamma^4}\right] - 6$$

$$\Rightarrow I = 6\, \zeta(4) - 6$$

$$\Rightarrow I = 6 \times \frac{\pi^4}{90} - 6.$$

$$\Rightarrow I = \frac{\pi^6}{15} - 6$$

$\zeta(4) = \frac{\pi^4}{90}$ can easily by Diverse by considering the Fourier - series expansion of x^2 on $(-\pi, \pi)$ of parseval identity.

Question - 27

By using the substitution $u = x + u\sqrt{x}$ in the integral definition of $\Gamma(x+1)$, show that for large values of n

$$n! \simeq \sqrt{2\pi}\, e^{-n}\, n^{n+1/2}, \quad n \in \mathbb{N}.$$

$$\boxed{\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt \quad \& \quad \Gamma(x+1) = \int_0^\infty t^{x} e^{-t}\, dt.}$$

Proof: let, $t = x + u\sqrt{x}$

$dt = \sqrt{x}\, du$

$t = 0 \quad , u = -\sqrt{x}$

$t = \infty \quad u = \infty.$

$$\Rightarrow \Gamma(x+1) = \int_{-\sqrt{x}}^{\infty} (x + u\sqrt{x})^x e^{-(x+u\sqrt{x})} \sqrt{x}\, du$$

$$\Rightarrow \Gamma(x+1) = \int_{-\sqrt{x}}^{\infty} x^x \left[1 + \frac{\sqrt{x}}{x} u\right]^x e^{-x} e^{-u\sqrt{x}} x^{1/2} du$$

$$\Rightarrow \Gamma(x+1) = x^{x+1/2} e^{-x} \int_{-\sqrt{x}}^{0} e^{-u\sqrt{x}} \left[1 + \frac{u}{\sqrt{x}}\right]^x du$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^{0} e^{-u\sqrt{x}} \times e^{\ln\left[(1 + u/\sqrt{x})\right]^x} du.$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^{0} e^{-u\sqrt{x}} \times e^{x\ln(1+u/\sqrt{x})} du.$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2} e^{-x}} = \int_{-\sqrt{x}}^{0} \exp\left[-u\sqrt{x} + x\ln\left(1 + \frac{u}{\sqrt{x}}\right)\right] du$$

- Now $\ln(1+w) = w - \frac{w^2}{2} + \frac{w^3}{3} - \frac{w^4}{4} + \dots \; -1 < w \leq 1$

So if $-\sqrt{x} < u \leq \sqrt{x}$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2}e^{-x}} \int_{-\sqrt{x}}^{0} \exp\left[-u\sqrt{x} + x\left[\frac{u}{\sqrt{x}} - \frac{u^2}{2x} + \frac{u^3}{3x^{3/2}} - \frac{u^4}{4x^2} + \dots\right]\right] du.$$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2}e^{-x}} = \int_{-\sqrt{x}}^{\sqrt{x}} \exp\left[-u\sqrt{x} + x\left[\frac{u}{\sqrt{x}} - \frac{u^2}{2x} + \frac{u^3}{3x^{1/2}} + \frac{u^4}{4x^2} + \dots\right]\right] du$$

$$+$$

$$\int_{\sqrt{x}}^{\alpha} \exp\left[-u\sqrt{x} + x\left[\frac{u}{\sqrt{x}} - \frac{u^2}{2x} + \frac{u^3}{3x^{3/2}} - \frac{u^4}{4x^2} + \dots\right]\right] du$$

- As $x \to \alpha$ the second integrate tends to zero, conf. the top units is α.

- As $x \to \alpha$ ~~&~~ Integrate in the first ~~Bet~~ integral Between $\exp(-u^2)$.

Thus $\frac{\Gamma(x+1)}{x^{x+1/2}e^{-x}} \approx \int_{-\alpha}^{\alpha} e^{-u^2} du$ As $x \to \alpha$

$$\Rightarrow \frac{\Gamma(x+1)}{x^{x+1/2}e^{-x}} \approx \sqrt{2\pi} \quad \text{As } x \to \alpha$$

$$\Rightarrow \Gamma(x+1) \approx \sqrt{2\pi}\, x^{x+1/2} e^{-x}. \quad \text{As } x \to \alpha.$$

- let. $x = n$ = positive integer.

$$\Gamma(n+1) \approx \sqrt{2\pi}\, n^{n+1/2}\, e^{-n}$$

$$n! \approx \sqrt{2\pi}\, e^{-n}\, n^{n+1/2}.$$

//.

A PROJECT REPORT

ON

"Study Of Center Of Gravity"

PAPER CODE:- MAT-HE-6086

IN PARTIAL REQUIREMENT OF FULFILMENT OF THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS,2023

Submitted by:-

Asraful Jakirin

G.U.Roll No:- US-201-003-0007

G.U.Registration No:- 20001732

Department Of Mathematics

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Asraful Jakirin
23/05/2023

23/05/2023
Associate Prof. & HoD
Deptt. of Mathematics
B.P. Chaliha College,Nagarbera

CERTIFICATE

Asraful Jakirin, a student of B.SC 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) bearing Roll No: **US-201-003-0007, Reg. No: 20001732** has submitted the project report entitled **"CENTER OF GRAVITY"** and appeared in viva-voce before the evaluation committee and presented the project report.

Evaluation Committee	Signature
1. Chairman / External	23/5/23
2. H.O.D.	23/5/23
3. Supervisor	23/5/23

CERTIFICATE

This is certified that **Asraful Jakirin**, a student of B.SC 6^{th} semester of B.P. Chaliha College in the Department of Mathematics (Honours) has carried out this project report entitled "**CENTER OF GRAVITY**" by himself. He has fulfilled all requirements for the preparation of project report for the 4th paper of B.Sc. (CBCS) 6th semester examination, 2023

He worked very hard to prepare this project report. I wish his very success in his life.

Date: 23/05/2023

Place: Nagarbera

23/5/23

Head of the Department

Associate Prof. & HoD
Deptt. of Mathematics
B.P. C. [illegible] College, Nagarbera

CERTIFICATE

This is certified that **Asraful Jakirin**, a student of B.SC 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) has carried out. This project report entitled **"CENTER OF GRAVITY"** by himself under my supervision. He has fulfilled all the requirements for the preparation of project report for the 4th paper of B.Sc. (CBCS) 6th semester examination, 2023

He worked very hard to prepare this project report.

I wish him very success in his life.

Date:.........23/5/23

Place:....~~Azara Path~~ Nayalun

Supervisor 23/5/23

ACKNOWLEDGEMENT

This project report entitled "**CENTER OF GRAVITY**" is intended for drawing attention of mathematical applications in the field of Partial Differential Equations.

My sincere thanks to professor **Dr. Atowar Rahman**, Head of the Department of mathematics of B.P. Chaliha College, Nagarbera for his guidance & providing necessary facelties to carry out my project report.

My sincere thanks to **Dr. Deepjyoti Borgohain**, Assistant Professor B.P. Chaliha College, Nagarbera for his help to carry out my project report successfully.

Asraful Jakirin

Asraful Jakirin

B.Sc. (CBCS) 6th semester

GU Roll No: US-201-003-0007

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B.P. Chaliha College, Nagarbera

Date: 23/05/203

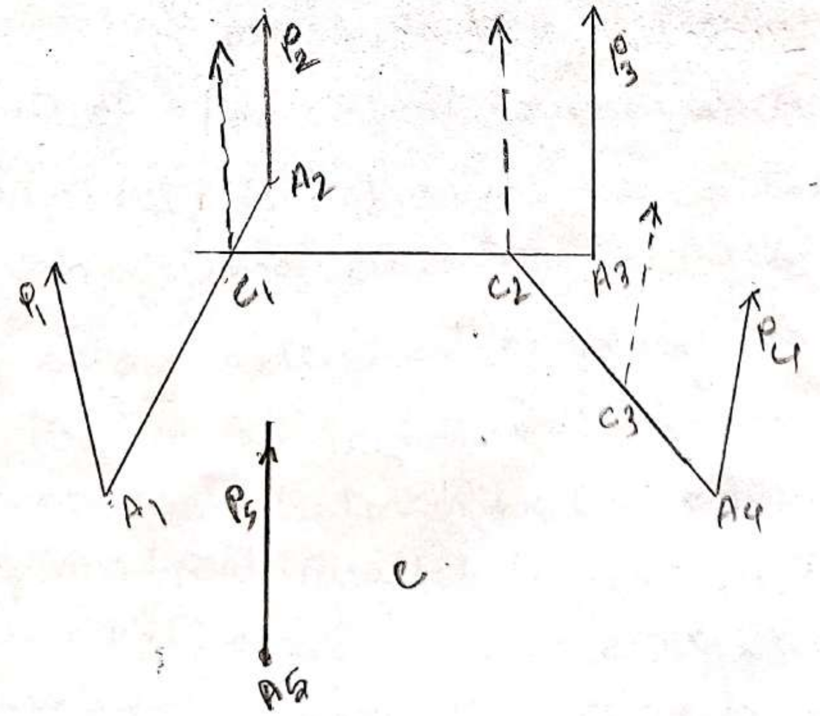
Place: Nagerbera

CENTRE OF GRAVITY

(a) Centre of like Parallel forces.

Let $P_1, P_2, P_3, \ldots$ be a set of like parallel forces acting on a rigid body at $A_1, A_2, A_3 \ldots$ respectively. Join $\overline{A_1A_2}$ and divide it internally at C_1 such that $A_1C_1 : A_1C_2 = P_2 : P_1$.
Then the resultant of P_1 and P_2 is a parallel force (P_1+P_2) acting at C_1. Join $\overline{C_1A_3}$, and divide it internally at C_2 such that $C_1C_2 : C_2A_3 = P_3 : (P_1+P_2)$.
Then the parallel force $(P_1+P_2+P_3)$ acting at C_2 is the resultant of (P_1+P_2) at C_1 and P_3 at A_3, that is, of P_1 at A_1, P_2 at A_2 and P_3 at A_3. Next join $\overline{C_2A_4}$, and divide it at C_3 such that

$$C_2C_3 : A_3A_4 = P_4 : (P_1+P_2+P_3)$$



Proceeding in this manner till all the forces are exhausted we finally arrive at a point c through which passes a force of magnitude ΣP and parallel to the system, which is the resultant of all the given parallel forces.

Now, it is ~~evid~~ evident that the positions of c_1, c_2, c_3, etc. depend only on the magnitudes of P_1, P_2, P_3 ------ and on the positions of the point A_1, A_2, A_3 ------ where they act, but have nothing to do with the common direction of the parallel forces.

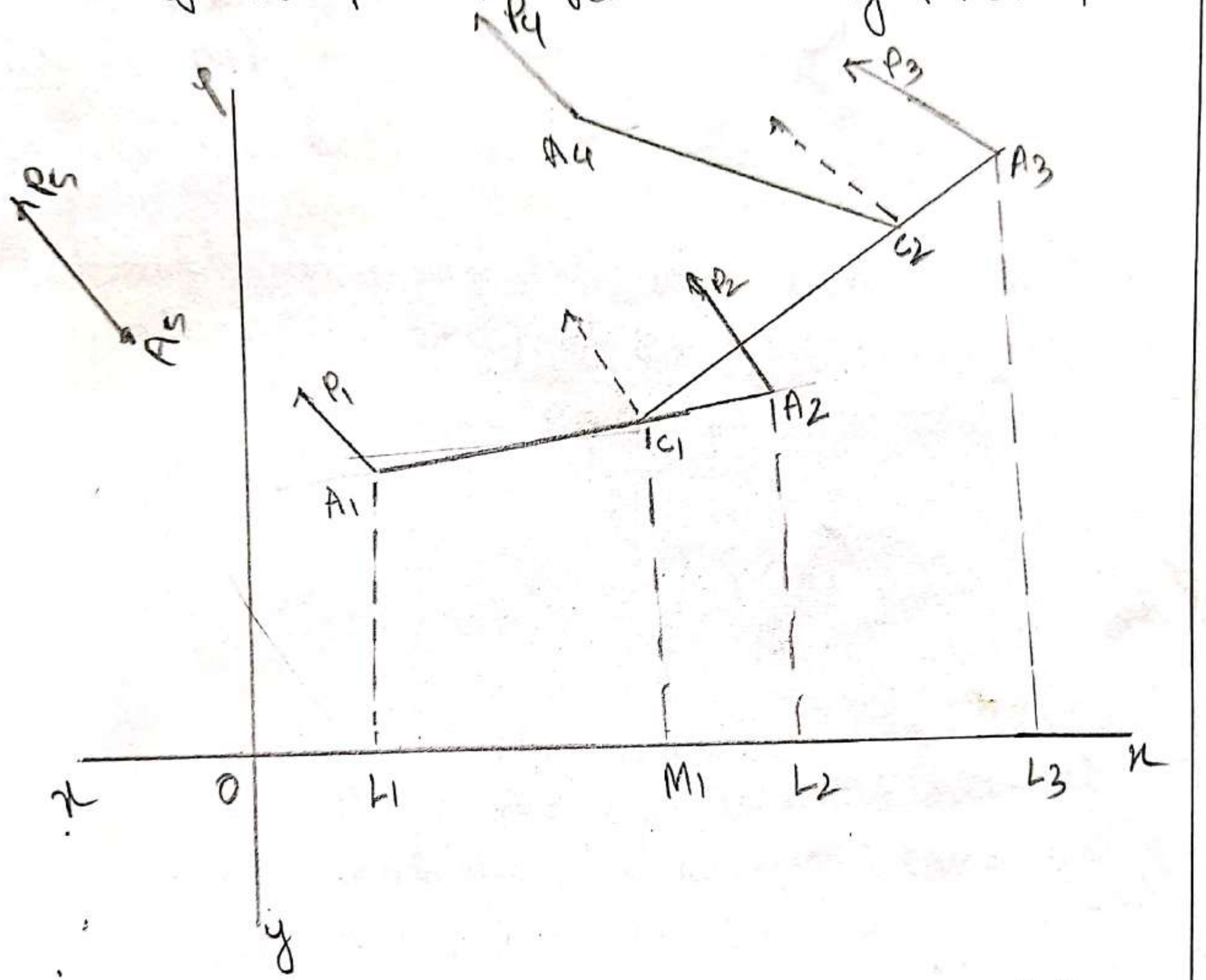
Hence the point c arrived at, through which the final resultant of the parallel forces passes, is fixed, whatever be the common direction of the parallel forces, so long as their magnitudes and points of application remain unchanged. This fixed point c is defined as the centre of the given system of like parallel forces.

We ~~my~~ may note that c being the point through which the resultant (magnitude ΣP) always passes, whatever be the common direction of the given ~~parll~~ parallel forces P_1, P_2, P_3, --- it is a unique point, and will come out to be the same ~~whath~~ whatever order we proceed to combine.

The given forces in succession.

Analytical determination.

We shall confie ourselves to the case of a set of like parallel forces acting in one plane.



Let P_1, P_2, P_3 ---- be a set of like parallel forces acting at A_1, A_2, A_3, ---- on a plane, and let $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, --- be the co-ordinates of A_1, A_2, A_3, ---- referred to a set of rectangular axes on the plane.

The resltant of P_1 at A_1 and P_2 at A_2 is (P_1+P_2) acting at C_1 on $\overline{A_1A_2}$ such that $A_1C_1 : C_1A_2 = P_2 : P_1$.

Let ξ_1, η_1 be the co-ordinates of C_1. Then $\overline{A_1L_1}$, $\overline{A_2L_2}$, and $\overline{C_1M_1}$ being perpendiculars upon OX, since these are are parallel lines,

$$\frac{L_2M_1}{M_1L_2} = \frac{A_1C_1}{C_1A_2}, \text{ that is } \frac{\xi_1 - x_1}{x_2 - \xi_2} = \frac{P_2}{P_1},$$

whence we get $\xi_1 = \dfrac{P_1x_1 + P_2x_2}{P_1 + P_2}$

Exactly in a similar manner, the x-co-ordinate (ξ_2, say) of C_2 where the resultant $(P_1 + P_2 + P_3)$ of $(P_1 + P_2)$ at C_2 and P_3 at A_3 act is given by

$$\xi_2 = \frac{(P_1 + P_2)\xi_1 + P_3x_3}{(P_1 + P_2) + P_2} = \frac{P_1x_1 + P_2x_2 + P_3x_3}{P_1 + P_2 + P_3} \quad \text{(1)}$$

Proceeding in this manner, when all the forces are exhausted the centre of the parallel forces through which the final resultant passes having co-ordinates ξ, η, we get

$$\xi = \frac{P_1x_1 + P_2x_2 + P_3x_3 + \cdots}{P_1 + P_2 + P_3 + \cdots} = \frac{\Sigma Px}{\Sigma P}.$$

and similarly.

$$\eta = \frac{P_1y_1 + P_2y_2 + P_3y_3 + \cdots}{P_1 + P_2 + P_3 \cdots},$$

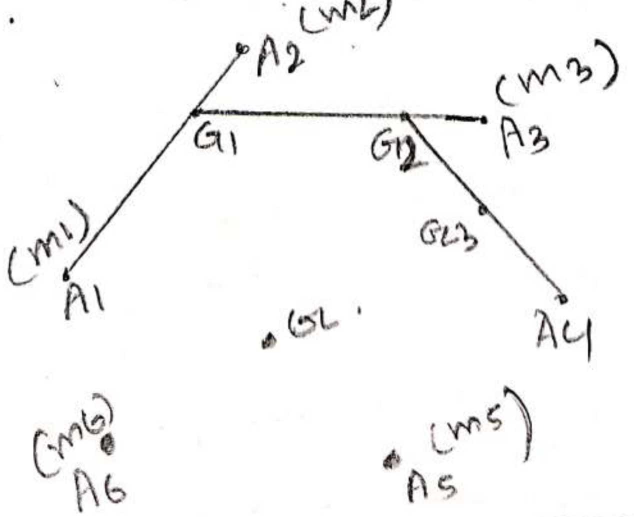
$$= \frac{\Sigma Py}{\Sigma P}.$$

Centre of mass.

Let a system of particles of masses m_1, m_2, m_3, ---- be placed at the point A_1, A_2, A_3 ----.

Divide $\overline{A_1A_2}$ at G_1 in the inverse ratio of the masses at its extremities, that is $A_1G_1 : G_1A_2 = m_2 : m_1$. Assume the total mass $(m_1 + m_2)$ be collected at G_1. Divide $\overline{G_1A_3}$ at G_2 in the inverse ratio of the masses at the extremities such that ~~$G_1G_2 : G_2A_3 = m_3$~~:

$G_1G_2 : G_2A_3 = m_3 : (m_1 + m_2)$, and assume the total mass at the extremities, that is $(m_1 + m_2 + m_3)$ to be concentrated at G_2. Proceeding in this manner, when all the particles have been exhausted, we arrive at a final point G which is defined as the centre of mass, or centre of inertia of the given system of particles.



The symmetry of the forms of the co-ordinates also shows that the point G is unique, and will be arrived at finally, in whatever order the masses may be combined in succession.

If instead of discrete particles, we have a finite body of any. may consider it as an agglomeration of an infinite number of infinitely small particles, and define its centre of mass as above. It will be a definite point in the body.

Centre of mean - position.

More generally, let there be a set of point $A_1, A_2, A_3, \ldots$ and let $m_1, m_2, m_3, \ldots$ be any set of numbers which are associated with these points respectively. Now join any two points $A_1; A_2$ and divide $\overline{A_1 A_2}$, at G_1 in the inverse ratio of the numbers associated at the ends such that $A_1G_1 : G_1A_2 = m_2 : m_1$. Assume the number $(m_1 + m_2)$ to be associated with G_1. Join $G_1 A_2$ and divide it at G_2 such that

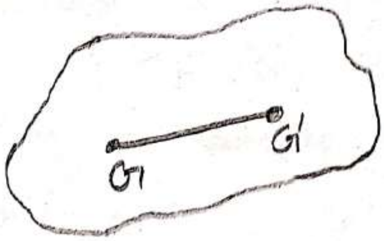
$$G_1G_2 : GA_3 = m_3 : (m_1 + m_2),$$

and suppose G_2, to be associated with $(m_1 + m_2 + m_3)$. Proceeding in this manner. we ~~find~~ finally arrive at a unique point G which is called the centre of mean - position of the given points for the system of given multipliers.

If the points $A_1, A_2 \ldots$ etc. are coplanar with co-ordinates $(x_1, y_1), (x_2, y_2)$, etc, we get exactly as in the previous article, the co-ordinates of G given by

$$\bar{x} = \frac{\Sigma mx}{\Sigma m}, \quad \bar{y} = \frac{\Sigma my}{\Sigma m}$$

The centre of gravity of a body is unique.



For, if possible, let a body have two centres of gravity, G and G' the weight of the body passes through both G and G', by definition, in whatever manner the body may be held. Now hold the body such that GG' is horizontal, the weight, which is a vertical force, cannot now pass through both G and G' unless they coincide. Hence, there cannot be two distinct centres of gravity of the body.

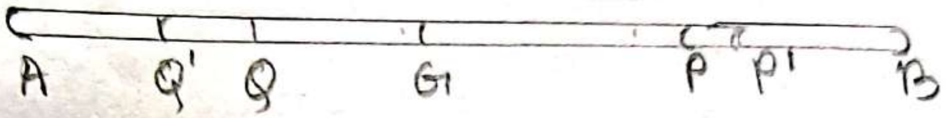
Contre of gravity.

We know from the law of gravitation that every material particle is attracted towards the centere of the earth with a force which is proportional to the mass of the particle, and this force we call its weigh.

Now, given any material body, we can consider it to be an assemblage of particles each of which is acted on by the earths attraction, and all these forces passing through the centre of the earth they have got a single resultant which we call the weight of the body. If the body be held in different positions, the positions of the lines of action of this weight relative to the body will be different. Now, in some cases, the line of action of the weight always passes throught a fixed point in the body us, however the body may be placed; for instance, in case of a spherical body by.

Determination of centre of gravity in Special cases.

I. A Thin uniform rod.

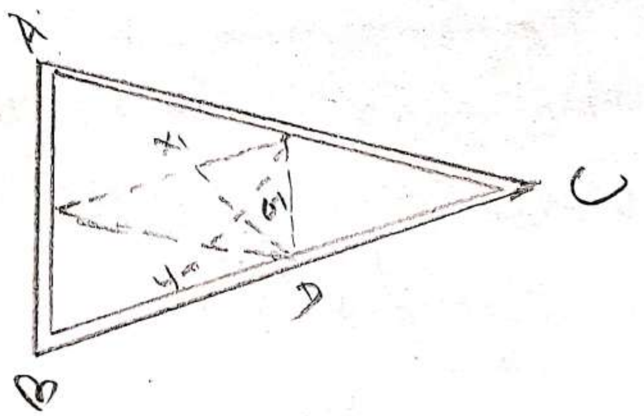


Let $\overline{AB}$ be a thin ~~unif~~ uniform rod of any material, G the middle point of $\overline{AB}$. Consider two equal infinitesimal lengths $\overline{PP'}$ and $\overline{QQ'}$ of the rod equidistant from G so that $GP = GQ$. Since the rod is uniform, the weights of these two elements (which can ultimately be treated as two particles equidistant from G) are equal, both vertically downwards, and the resultant of these two equal and like parallel forces acts at the middle point G.

Since $AG = GB$, the whole rod can be divided into pairs of such equal infinitesimal elements equidistant for from G, and for each pair the resultants of the weights act at G. Hence, the weight of the whole rod acts at the middle point G.

Three rods forming a triangle.

Let $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ be three thin uniform rods of same material and thickness (or parts of a thin uniform wire) foring a triangle ABC. Let D, E, F be the middle points of $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ ~~as~~ respectively. The weights of the uniform rods $\overline{AB}$; $\overline{AC}$ act vertically downwards at their middle points F and E. ~~resp~~ respectively, their



magnitudes being proportional to their lenghts. The resultant of these two weights which are like parallel farcess, acts at a point X on $\overline{FE}$ where.

$$\frac{FX}{XE} = \frac{\text{weight at } E}{\text{weight } F} = \frac{\text{length } AC}{\text{length } AB} = \frac{2DF}{2DE} = \frac{DF}{DE}.$$

Thus $\overline{DX}$ bisects the angle EDF.

Now, the resltant of the weight of the rod $\overline{BC}$ acting at its middle point D and the joint weight of the ~~rod $\overline{BC}$ acting at its~~ $\overline{AB}$ and $\overline{AC}$ acking at x . will act at ~~sme~~ some point on $\overline{DX}$. Hence, the combined C.G of the three rods is situated on the line $\overline{DX}$.

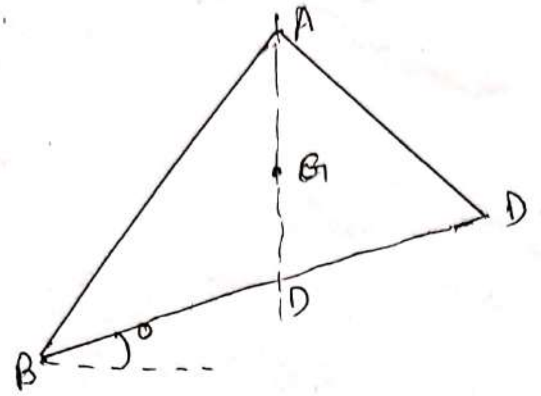
~~Similar~~ similarly, combining the weights $\overline{AB}$, $\overline{BC}$ first and then considering the weight of $\overline{AC}$, the combined C.G. of the three rods is shown to be some point on $\overline{EY}$ which bisects the angle DEF.

Thus, the combined C.G. of the system of three rods, being a common point situated ~~on~~ on both $\overline{DX}$ and $\overline{EY}$, is their point O:

intersection, that is, the required C.G. is the in-centre of the triangl.

DEF formed by joining the middle points of the rods.

Ex. Triangular lamina ABC hangs at rest, one of the angles A being supported at a fixed point. Find the angle which the lower side makes with the horizon.



D being the mid-point of the lower side $\overline{BC}$, the centre of gravity G of the triangle lies on $\overline{AD}$. Now, as the lamina hangs in equilibrium under the weight acting vertically downwards through G and the reaction at the point of support A, these two forces must be equal and opposite, acting in the same ~~to~~ straight line. Thus $\overline{AGD}$ must be vertical. If θ be the required inclination $\overline{BC}$ to the horizon, then.

$$m\angle ADC = 90^\circ - \theta$$

Now, $\because BD = DC$, $\therefore$ Therefore $\frac{BD}{AD} = \frac{DC}{AD}$

or, $\dfrac{\sin BAD}{\sin ABD} = \dfrac{\sin CAD}{\sin ACD}$

That is, $\dfrac{\sin(90^\circ - \theta - B)}{\sin B} = \dfrac{\sin(\theta - C)}{\sin C}$

or. $\dfrac{\cos(\theta + B)}{\sin B} = \dfrac{\cos(\theta - C)}{\sin C}$

Therefore $\cos\theta \cot B - \sin\theta = \cos\theta \cot C + \sin\theta$

Therefore $\tan\theta = \frac{1}{2}(\cot B - \cot C)$

$\therefore \quad \theta = \tan^{-1}\left\{\frac{1}{2}(\cot B - \cot C)\right\}$

A PROJECT REPORT

ON

"Study Of Propositional Logic"

DEPT. OF MATHEMATICS B.P. CHALIHA COLLEGE Estd.1974 NAGARBERA

PAPER CODE:- MAT-HE-6086

IN PARTIAL REQUIREMENT OF FULFILMENT OF THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS,2023

<u>Submitted by:-</u>

JAHIRUL ISLAM

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Jahirul Islam
23/05/2023

CERTIFICATE

Jahirul Islam, a student of B.Sc (CBCS) 6th semester of B.P. Chaliha College in the department of Mathematics (Honours) bearing GU Roll No:US-201-003-0019,Reg No:20001744 has submitted the project report entitled "PROPOSITIONAL LOGIC"and appeared in viva-voce before the evaluation committee and presented the project report.

Evaluation committee	signature
1.Chairman / External	23/5/23 (A.B. Siddique).
2.H.O.D.	Atowar Rahman. 23/5/23
3.Supervisor	Bordoloi 23/05/23

CERTIFICATE

This is certified that Jahirul Islam,A student of B.Sc. 6th semester of B.P. Chaliha College in the Department of Mathematics (Honours) has carried out this project report entitled "PROPOSITIONAL LOGIC" by himself under my supervision.He has fulfilled all the requirements for the preparation of project report for the 4th paper of B.Sc.(CBCS) 6th semester examination,2023

He worked very hard to prepare this project report.

I wish him very success in his life.

23/05/23

Supervisor

Date:.23/05/2023

Place:......Nagarbera...

CERTIFICATE

This is certified that Jahirul Islam , A student of B.Sc 6th semester of B.P Chaliha College in the Department of Mathematics (Honours) has carried out this project report entitled"PROPOSITIONAL LOGIC" by himself. He has fulfilled all requeirements for the preparation of project report for the 4th paper of B.Sc.(CBCS) 6th semester examination,2023

He worked very hart to prapare this project report.

I wish his very success in his life.

Atowar Rahman.

Date:..23/05/2023

place :..........Nagarbera

Head of the Department
Associate Prof. & HoD
Deptt. of Mathematics
B.P. Chaliha College,Nagarbera

ACKNOWLEDGEMENT

This project report entitled " PROPOSITIONAL LOGIC" is intended for drawing attention of mathematical applications in the field of Algebra

My sincere thanks to professor Dr.Atower Rahman,Head of the department of mathematics of B.P. Chaliha College,Nagarbera for his guidance and providing necessary facilities to carry out my project report.

My sincere thanks to Dr.Deepjyoti Borgohain,Assistant Professor,B.P. Chaliha College,Nagarbera for his help to carry out my project report successfully.

Jahirul Islam

Jahirul Islam

B.Sc.(CBCS) 6th semester

GU Roll No:US-201-003-0019

Reg No:20001744

B.P. Chaliha College,Nagarbera

Date:..23/05/23

Place:.Nagarbera

PROPOSITIONAL LOGIC

INTRODUCTION-

Logic is the basic of all mathematical reasoning and of all automated reasoning. The rules of logic specific the meaning of mathematical statement. These rules help us understanding and reason with a statement such as-

there exist x such that $x \neq a^2 + b^2$ where x, $a, b \in \mathbb{Z}$. which is simple english means "there exists an integer that is not the sum of two squares." The logic provides, basic for many area of computer science such as digital logic designing and compitability. "propositional logic also known as sentential logic and statement logic. is the branch of logic that studies ways of joining and modifying entire propositions. statement or sentence to form more complicated propositions, sentences or statements as well as the logical relationships and properties that are derived.

1.1 BASIC CONCEPT OF PROPOSITION OR STATEMENT.

1.1.1 PROPOSITION (OR STATEMENT)

In mathematics, one deals with a large number of sentences which are strings of words and symbols having precise meanings. All such sentences are classified as being true or false and they are called statements. Thus, a statement is either true or false; never both or in between. For examples "Milk is white' and 'Assam is the capital of India'. The first sentence is true and is a statement, while the second statement is false. Basically, a statement expresses a judgement or opinion.

The truth value of proposition is true and it denoted by 'T', on the other hand the false value of a preposition is false and it is denoted by 'F'.

For example.

(i) 5 is a prime number.

(ii) $7+3=10$

(iii) $5+3=9$

(iv) Moscow is the capital of china.

All the above sentences are proposition or statement where the first and second statement are true and third and fourth are false.

The sentence: "the sentence I am reading is false. it is not a statement because it can be neither true or false.

A sentence can be a statement even if you do not Know decisively whether it is true or false, or even if its being true or false depends on the context. For example—

(i) Every even integer ≥ 4 is a sum of two primes.

(ii) $x+3=7$.

(iii) The number of students in the class is fifty.

However, it is not yet Known whether sentence (i) is true or false. The sentences (ii) and (iii) are context specific; they are true or false depending on the situations. (ii) is true if $x=4$ but false if $x=3$. Nevertheless, the above sentences are statement.

NOTE: It is customary to represent simple statement by p, q, r..... Known as proposition variable. propositional variable can only assume two value true or false. These are two propositional constants. T and F that represents true or false respectively.

1.1.2 COMPOUND PROPOSITION

To represent propositions, propositional variables are used. By convention, these variables are represented by small alphabates such as the area of logic which deals with propositions is called propositional calculus or propositional logic. propositions constructed using one or more proposition are called compound statement.

consider the statement: "The boy is intelligent and handsome." Notice that it is consist of two statements joined by "and." the statement may be read as "The boy is intelligent and the boy is handsome."

A statement of this kind is called a conjunction of statements.

1.1.3 CONNECTIVES.

The operational symbols $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$ are called connectives. From now on we shall assume that the letters p, q, r ----- with or without subscripts, denoted variable over the set of statements. We call them statement letters.

The following are used to represent connectives-

symbols used	connectives word	Nature of the compound statement formed by connectives	symbolic form
$\sim, -$	not	Negation	$\sim p$
$\wedge$	and	conjunction	$p \wedge q$
$\vee$	or	Disjunction	$p \vee q$
$\Rightarrow, \rightarrow$	if ---- then	Implication	$p \Rightarrow q$
$\Leftrightarrow, \leftrightarrow$	if and only if	Equivalence	$p \Leftrightarrow q$
$\underline{\vee}, \oplus$	XOR	'Exclusive OR' (OR XOR)	$p \underline{\vee} q$
$\downarrow$	NOR	Negational of disjunction	$p \downarrow q$
$\uparrow$	NAND	Negation of consjunction	$p \uparrow q$

1.2 OPERATION ON STATEMENT

The operations are truth functional in the sense of that truth value of the new statement are determined by the truth value of the component statement.

1.2.1 NEGATION

For any given statement p, there is another statement which is defined to be the statement that is true when p is false, and false when p is true. This statement is called the negation of p and is denoted by not-p or $\sim p$, which when translated to simple english means 'It is not the case that p or simply 'not p;

The truth value of $\sim p$ is the opposite of the truth value of p.

The negation of the statement "5+3=9 is $5+3 \neq 9$, and not the statement "5+3 = 8.

The truth table of $\sim p$ is —

p	$\sim p$
T	F
F	T

1.2.2 CONJUNCTION

For any two propositions p and q, their conjunction is denoted by $p \wedge q$, which means 'p and q'. The conjunction $p \wedge q$ is true when both p and q are true, otherwise false.

The truth table of $p \wedge q$ is-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

1.2.3 DISJUNCTION

For any two propositions p and q their disjunction is denoted by $p \vee q$, which means 'p or q'. The disjunction $p \vee q$ is true when either p or q is true otherwise false. The truth table of $p \vee q$ is-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

1.2.4. CONDITIONAL (OR IMPLICATION)

For any two propositions p and q, the statement 'if p then q' is called a conditional $p \to q$, p is called the hypothesis and q is called the conclusion or consequence. The implication is false when p is true and q is false otherwise it is true.

The truth table of $p \to q$ is–

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

1.2.5: BICONDITIONAL (OR- BI-IMPLICATION)

The operation 'bi- conditional' combines two statement p, q to form a new statement 'p if and only if q' denoted by $p \leftrightarrow q$. $p \leftrightarrow q$ has the same truth value as $(p \to q) \wedge (q \to p)$. The implication is truth when p and q have same truth value and is false otherwise.

The truth table of $p \leftrightarrow q$ is –

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

1.2.6 "EXCLUSIVE OR" OR "XOR"

For any two propositions p and q their exclusive or is denoted by $p \oplus q$ which means "either p or q but not both". The exclusive or $p \oplus q$ is true when either p or q is true and false when both are true or false.

The truth table of $p \oplus q$ is –

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

1.27. DERIVED CONNECTIVES.

NOR:

It means negation of disjunction of two statements. NOR of p and q is true when both p and q are false. Denoted by $p \downarrow q$ its truth table is —

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

NAND:

It means negation of conjunction of two statements. NAND of preposition p and q is false when both p and q are true, otherwise ~~true~~.
It is denoted by $p \uparrow q$ and $p \uparrow \equiv \sim(p \wedge q)$

Its truth table is as follows —

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	F

1.2.8 STATEMENT FORMULA

A statement formula is defined recussively as follows –

(i) Any statement letter is a statement formula.

(ii) If A and B are statement formula then the expression $\sim A$, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B$ are also statement formula.

(iii) Only those expression are statement formula which are considered by using (i) and (ii).

1.2.9 TRUTH FUNCTION.

For every assignment of truth values T OR F to the statement letters occuring in a statement formula their corresponds by virtue of the truth table for the connectives, a truth table for the statement formula. Thus each statement formula containing n distinct statement letter determines a function called truth function i.e; a function from $\{T, F\}^n$ into $\{T, F\}$, which can be graphically represented by a truth table for the statement formula.

1.2.10 TAUTOLOGY

A statement formula is called a tautology if in its truth table the column under the statement formula contains only T's i.e; if its truth functions takes always the value T. We generally use the notation '⊨' to indicate that A is a tautology. ("⊨" is known as Kleene's symbol).

for example: $\sim(p \wedge q) \vee q$ is a tautology.

1.2.11 CONTRADICTION

A statement formula is called a contradiction if in its truth table the column under the formula containing only F's i.e; if its truth function takes always the value F's.

for example: $p \wedge (q \wedge \sim p)$ is a contraddiction.

NOTE: A statement formula which is neither a contradiction nor a tautology is called contingent.

1.2.12 LOGICAL CONSEQUENCE.

If $A \to B$ is a tautology then A is said to logically imply B, or alternatively, B is said to be logical consequence of A. for example: $p \wedge (p \to q) \to q$ is a tautogy.

1.2.13 LOGICAL EQUIVALENCE:

If $A \leftrightarrow B$ is a tautology then A and B are said to be logically equivalent and we denote this fact by $A \equiv B$. observe that $A \equiv B$ if and only they have the same truth table.

for example: $p \to q \equiv \sim p \vee q$.

1.3 LAWS OF PROPOSITIONAL LOGIC

By truth tables, using the principle of substitution we can easily varify that the following laws are valid in the set of statement formulas. Let A, B, C be any three statement formula, then

(i). IDEMPONENT LAW:

$$A \vee A \equiv A$$

$$A \wedge A \equiv A$$

(ii). ASSOCIATIVE LAW:

$$(A \vee B) \vee C \equiv A \vee (B \vee C)$$

$$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$$

(iii). COMMUTATIVE LAW:

$$A \vee B \equiv B \vee A$$

$$A \wedge B \equiv B \wedge A$$

(iv). DISTRIBUTION LAW:

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

(v). DE- MORGAN'S LAW:

$$\sim (A \vee B) \equiv \sim A \vee \sim B$$

$$\sim (A \wedge B) \equiv \sim A \vee \sim B$$

(vi). ABSORPTION LAW:

$$A \vee (A \wedge B) = A$$

$$A \wedge (A \vee B) = A$$

(vii). CONTRAPOSITIVE LAW:

$$A \rightarrow B \equiv \sim B \rightarrow \sim A$$

(viii). IDENTITY LAW:

$$A \vee F \equiv A$$

$$A \vee T \equiv T$$

$$A \wedge T \equiv A$$

$$A \wedge F \equiv F$$

where T and F denotes respectively a tautology and a contradiction.

(ix). COMPLEMENT LAWS:

$$A \vee \sim A \equiv T$$

$$A \wedge \sim A \equiv F$$

$$\sim T \equiv F$$

$$\sim F \equiv T.$$

(x). INVOLUTION LAWS:

$$\sim(\sim p) \equiv p.$$

REFERENCES:-

1. A Foundation Course in MATHEMATICS by A. Kumar, S. Kumaresan and B.K. Sarma.

2. Introduction to discreate Mathematics by M.K Sen and B.S Chakraborty.

3. A Textbook of Discreate Mathematics by Dr. Swapan Kr. Sarkar.

4. Internet.

CERTIFICATE

Certified that **TASLIMA NASRIN**, a student of B.Sc. final year ofdepartment of Zoology of Bimala Prasad Chaliha College, Nagarbera has worked out his field project work, entitled studies on “Bird nest building and Bird Behavior” in Nagarbera Area (Nagarbera) under my supervision.

Sd/-
16/5/23

(Tapan Kr. Goswami)

Sign. of Supervisor.

CONTENTS

INTRODUCTION:

The Brahmaputra drainage system in North-East India is one of the largest hydrographic basin. Southest Asia and sustains very rich and diverse aquatic gene pole , particularly fishes and such as the region is featured among the global hotspot of fresh water fish biodiversity. All north eastern states have handful resources of fishes as well as other aquatic species in terms of many rivers with their tributary's streams, rivulets,wetlands,lakes,ponds,tank etc. Assam has vast and varied fresh water resources and it being regarded as one of the richest spots of biodiversity in India.

Fish invariably one of the most important biotic components of an aquatic ecosystem which apart from forming protein rich food source for human beings,also act as a good bio indicator of a water body.The Northeastern region of India is one of the hot spots of fresh water fish biodiversity in the world. However,the rich biodiversity of the freshwater fish of the India has been rapidly declining over the year due to excessive human activities and as well as another environment factor, the loss of biodiversity and its effect are predicted to be greater for aquatic ecosystems than for terrestrial ecosystems Sala et al.(2000) .

CONCLUSION:

In conclusion documentation of biodiversity of fish is one of the most important aspects of study from ichthyological point of view. No fishing should be allowed during spawning season especially with mosquito nets for sustainable ecosystem management as well as to sustain the identified fish fauna in the beel. People should be aware about the threatening factors responsible for declining of fish fauna of the river .

CERTIFICATE

Certified that **RASHIDA AHMED**, a student of B.Sc. final year ofdepartment of Zoology of Bimala Prasad Chaliha College, Nagarbera has worked out his field project work, entitled studies on "Bird nest building and Bird Behavior" in Nagarbera Area (Nagarbera) under my supervision.

Gosw.
17/5/23

(Dr. Tapan Kr. Goswami)
Signature of Supervisor

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15/05/2023

Dr. Tapan Kumar Goswami

Signature

CONTENTS

INTRODUCTION

Nest building is a taxonomically widespread activity, with birds, mammals, reptiles, fish, and insects all constructing nests of some description in which to lay eggs and/or raise offspring. There is a huge amount of variation in nest design across taxa, with nests varying from underground burrows dug by mammals, minimal nest scrapes on the ground in which game birds lay their eggs, the craters constructed by fish on the bed of water bodies, the huge mounds constructed by termites through to the cup-shaped nests of song birds in trees and bushes. Even within taxa, there is a great deal of variation in nest design and in birds, nests range from the small but elaborate cup-shaped nests built by passerine birds through to the huge mounds built by megapodes.

Nest design varies considerably between and even within taxa, yet all nests have the same basic, minimal, function which is to provide a receptacle in which animals can lay their eggs and/or raise their developing offspring. This rather simplistic view of nests has generally prevailed over the years, and studies examining the function of birds' nests have been scarce, particularly when compared to other stages of reproduction. Illustratively, a study of six commonly studied nestbox-breeding passerine birds showed that fewer than 6% of 676 published studies reported any aspect of nest characteristics, despite researchers using nestbox-breeding birds as model systems due to the ease with which their reproductive parameters can be quantified. This oversight is unfortunate as there is considerable evidence that nests are sophisticated structures that require considerable cognitive abilities to construct. Fortunately, our understanding of the design and function of birds' nests has increased considerably in recent years, and here, we review the functions of birds' nests. We begin by examining the influence of natural and sexual selection, before examining the influence of parasites and environmental variation in determining nest-building behaviors and nest design.

FIGURE : MAP OF NAGARBERA

Scanned with OKEN Scanner

Discussion

Overall, our meta-analysis did not detect any strong relationship between urbanization and nest mortality. However, predation rates of artificial nests and of natural nests showed opposite trends in their relationships with urbanization: namely, the chance of natural nests to fail tended to decrease (i.e., the odds of survival of natural nests increased by 33%) with increasing urbanization, whereas the trend in artificial nests was significantly different, and in the opposite direction (i.e., the odds of survival was about 35% lower in more urbanized habitats). None of our considered moderators, ecological or methodological, explained the variation in effect sizes consistently in either study type, although our sensitivity analyses suggest nest openness may have an effect (Table S11, Supplementary Discussion S3.1).

The weak positive trend for a correlation between nest survival and urbanization in natural nests is in line with the "predator-safe zone hypothesis" (Gering and Blair, 1999; Ryder et al., 2010), which assigns the lower predation risk in cities as a principal reason of why certain prey species can thrive in urban habitats. Lower predation rates can be the result of low abundance of nest predators (Møller, 2012). However, many potential nest predator species are found in higher abundances in cities than in the surrounding natural habitats (Jokimäki and Huhta, 2000; Haskell et al., 2001). This apparent contradiction is called the "nest predation paradox" (Fischer et al., 2012). This paradox might be resolved in several ways. First, some prey species are often extremely abundant and/or found in higher densities in more urban habitats (McKinney, 2006). Thus, despite the high absolute numbers of nest predators, their relative abundance to prey can still be low compared to natural habitats (Fischer et al., 2012). Second, urbanization changes predator species composition (Rodewald and Kearns, 2011). This shift can mean a decrease in specialized ("strong") nest predators and an increase in opportunistic ("weak") nest predators (Stracey, 2011). For example, snakes, which are often specialized nest predators, are less abundant in cities than in rural areas (Patten and Bolger, 2003). In contrast, house cats, which are both fed by humans and hunt for live prey, and thus are mostly opportunistic nest predators, become more abundant with housing density (Sims et al., 2008), although they might be less common (or less often outdoors) in city centers compared to suburbs. Third, potential predators may specialize on different prey in urban than in rural areas as the most abundant prey species might differ between them, which can relax predation pressure on less abundant species (Fischer et al., 2012). Alternatively, in the case of omnivorous predator species, the shift can be toward anthropogenic food sources that are easier to access, which can relax the actual predation pressure on all prey species (Rodewald et al., 2011). Finally, higher abundance of nest predators in urban habitats can facilitate local adaptation in the behavior of their prey. As high predation pressure in cities should eliminate those individuals that could not effectively defend their nests from predators (either by hiding their nests or actively mobbing predators), the current urban prey population is better at nest defense and thus has higher survival rates than rural prey (Stracey, 2011).

In contrast to natural nests, we found a decreasing trend in the survival of artificial nests with increasing urbanization. This result is in line with a meta-analysis which revealed more predation on artificial nests with increasing fragmentation of forest habitats (Hartley and Hunter, 1998). Also, similar to our findings, a number of studies showed that predation rates in artificial nests often do not reflect those observed in natural nests (Haskell, 1995; Weidinger, 2001; Moore and Robinson, 2004; Robinson et al., 2005), although one study that used artificial and natural nests in the same conditions did not find significant difference between the two (Blair, 2004). The discrepancy between artificial and natural nests may be explained by several different mechanisms. First, our sensitivity analyses on natural nests indicated that predation rates of cavity nests and open nests change differently with urbanization (Table S11). Namely, cavity nests are predated significantly less in urban than in rural habitats, while open nests show no such habitat difference. Since the majority of artificial nest studies were conducted on open nests, their results are more comparable to those from natural open nests than to those from hole-like nests (which are also likely to be located high above the ground and exposed to different conditions). Second, as mentioned above, it is hypothesized that local adaptation may make urban bird parents better at nest defense behavior than their non-urban conspecifics (Stracey, 2011). As artificial nests are not defended by parents, they may be more likely to be depredated in urban habitats than natural nests. Third, the local adaptation hypothesis also suggests that urban birds are better at hiding their nests from predators (Stracey, 2011). As the locations of artificial nests are chosen by the experimenters, rather than the birds themselves, nests may be placed in more conspicuous places in urban habitats, thus predators can find them more easily, resulting in a higher predation rate. Fourth, although artificial nests try to emulate natural nests as much as possible, they might still be perceived by predators as novel compared to natural nests. It has been hypothesized that urban animals show less food neophobia (i.e., are more likely to accept novel food sources) than their rural conspecifics (Sol et al., 2011), and thus non-urban predators may be aversive toward artificial nests. Fifth, some discrepancy between the results of natural vs. artificial nest studies may come from the fact that partially predated nests were counted as survived in most natural nest studies, whereas they were counted as predated in artificial nest studies. Partial predation may be more common in cities where the predator is more likely to be interrupted by human disturbance while feeding on a nest. Finally, we cannot exclude the possibility that the real predation pressure on natural nests is higher in cities than in rural habitats, as studies on artificial nests suggest, but the effect is masked by sampling bias. Urban natural nests that were not concealed or defended well-enough may already have been predated before the researchers found them, and thus not included in the sample, while this might not be the case in rural habitats. The resulting sampling bias may lead to a seemingly higher predation rate in rural compared to urban habitats.

In conclusion, our results show that natural nests tend to be less predated in urban habitats than in rural habitats, but this trend is not reflected by studies on artificial nests. We have several recommendations for future studies addressing the relationship of urbanization and nest predation. First, we suggest that the cause of nest failure should be identified as precisely as possible, because both predation and other forms of mortality (weather, nest abandonment, vandalism by humans) can vary with urbanization, affecting the overall nest survival. Second, to identify whether variation is due to differences between species, multiple populations of the same species should be studied, and studies should preferably include data from multiple species. Third, data from the currently available literature is not sufficient for more sophisticated analyses, such as testing multiple biological and ecological factors in the same model and possible interactions between them, therefore more studies, with more balanced design for ecological characteristics such as nest height and nest openness, are required. Finally, as patterns derived from studies using artificial nests often do not qualitatively reflect those derived from natural nests, researchers should perform studies where artificial and natural nests with similar characteristics are monitored within the same area (e.g., Blair, 2004), and investigate sampling bias. Such validation is important to draw firm conclusions regarding the level of predation in urban vs. rural areas in future studies.

The reasons for the decline of the sparrow population are loss of habitat due to rapid urbanisation, diminishing ecological resources for sustenance, high levels of pollution and emissions from microwave towers.

প্ৰকল্প (PROJECT)

SEC COURSE

শিৰোনাম: ভাৰতবৰ্ষৰ বিধেয়ক গৃহীতকৰণ প্ৰক্ৰিয়া

জমাকৰ্তা: বিজুস্মিতা কলিতা

শ্ৰেণী: তৃতীয় ষাণ্মাসিক

ৰোল নং: ৫৯

Certificate

This is to certify that Miss Bijusmita Kalita has worked under my Guidance and supervision for her project work on SEC COURSE, which is being submitted to the Dept. of political science.

Date : 28/05/2023

Place: Nagarbera

(Dr. Diganta Kalita)
Asst. professor

বিষয়সূচী
(content)

পৃষ্ঠা নং

3. ভাৰতবৰ্ষত বিধিবদ্ধ দুৰীকৰণৰ প্ৰক্ৰিয়াটো আলোচনা কৰা।

উত্তৰ :-

আৰম্ভণি :

প্ৰত্যেক ৰাষ্ট্ৰৰ সংবিধানত ৰাষ্ট্ৰখনৰ মৌলিক নীতিবোৰ লিপিবদ্ধ থাকে। সংবিধানখনৰ যোগেদি সেই নীতি আদৰ্শবোৰ বাস্তৱত ৰূপ দিয়া হয়। ভাৰতীয় সংবিধান অনুসৰি গুৰুত্বপূৰ্ণ কাৰ্যবোৰেই হল আইন প্ৰণয়ন কৰা। সংবিধানে ভাৰতীয় আইন প্ৰণয়নৰ সকলো দায়িত্ব কেন্দ্ৰীয় আইন সভা আৰু ৰাজ্যিক আইন সভাৰ হাতত অৰ্পণ কৰিছে। কেন্দ্ৰীয় সংসদে কেন্দ্ৰীয় তালিকাভুক্ত বিষয়সমূহ আৰু প্ৰয়োজনবোধে ৰাজ্যিক তালিকাভুক্ত বিষয়সূচীৰ ওপৰতো আইন প্ৰণয়ন কৰিব পাৰে। তাৰোপৰি সমৱৰ্তী সূচীৰ বিষয় আৰু অৱশিষ্ট ক্ষমতাৰ বিষয়ৰ ওপৰতো কেন্দ্ৰীয় সংসদে আইন প্ৰণয়ন কৰিব পাৰে।

② বিধেয়কৰ পৰিচয় :→

সংসদৰ অনুমোদনৰ কাৰণে আইনৰ যি প্ৰস্তাৱ উত্থাপন কৰা হয় তাক বিধেয়ক বা বিল বোলে। গতিকে বিধেয়ক হ'ল সংসদত প্ৰস্তাৱিত আইন এখনৰ খচৰা অৰ্থাৎ আইন পাণ্ডুলিপিক বিধেয়ক বা বিল বোলা হয়। বিল এখনক সাংবিধানিক প্ৰক্ৰিয়া অনুসৰি গৃহীত হোৱাৰ পিছতহে আইনত পৰিণত হয়। এইখন বিধেয়ক এটা নিৰ্দিষ্ট শিৰোনামেৰে আৰম্ভ হয়। বিধেয়ক এখনত এটা প্ৰস্তাৱনা আৰু কাৰ্যকৰণৰ সূত্ৰ থাকে। তদুপৰি বিধেয়ক এখনৰ প্ৰয়োগযোগ্যতাৰ পৰিমাণ দিয়া হয় আৰু কিছুক্ষেত্ৰত ই আইনত পৰিণত হোৱাৰ তাৰিখো নিৰ্দিষ্টকৈ দিয়া হয়।

কেন্দ্ৰীয় বিধানমণ্ডল বা সংসদৰ আটাইতকৈ গুৰুত্বপূৰ্ণ কাৰ্যটোৱেই হৈছে দেশৰ প্ৰশাসন কাৰ্য পৰিচালনা কৰাৰ ক্ষেত্ৰত প্ৰয়োজন হোৱা আইন সমূহ প্ৰনয়ন কৰা। যাৰ আধাৰত দেশৰ কাৰ্যপালিকাই প্ৰশাসন কাৰ্য সুচাৰুৰূপে পৰিচালনা কৰাৰ প্ৰয়াস কৰে। এই প্ৰশাসন কাৰ্য সু-শৃংখলাৰে পৰিচালনা কৰাৰ প্ৰয়াস কৰে এই প্ৰশাসনকাৰ্য সু-শৃংখলাৰে পৰিচালনা কৰাৰ বাবে প্ৰয়োজন হোৱা এই আইন সমূহক আৰম্ভণিতে এক খচৰাবিধি অথবা প্ৰস্তাৱ আকাৰে অনুমোদনৰ বাবে সংসদ উত্থাপন কৰা হয়।

অৰ্থাৎ আইনৰ এই খচৰাবিলি বা প্ৰস্তাৱসমূহকে আৰম্ভণিত বিল বা বিধেয়ক হিচাপে বিবেচনা কৰা হয়। এই বিল বা বিধেয়কত নিৰ্ধাৰিত প্ৰক্ৰিয়াৰ মাজেৰে সংসদে অনুমোদন জনালে আৰু ৰাষ্ট্ৰপতিয়ে যদি তাত সন্মতি প্ৰদান কৰে তেতে হে আইনলৈ ৰূপান্তৰিত হয়। এইবোৰ হৈছে বিবেচ্যমান বৈধানিক প্ৰস্তাৱ যিবিলাক বিল অথবা বিধেয়ক ৰূপত সংসদৰ মজিয়াত উত্থাপন কৰা হয়।

সাধাৰণতে বিল অথবা বিধেয়কসমূহক বহলভাৱে চৰকাৰী বিধেয়ক আৰু বেচৰকাৰী বিধেয়ক দুটা ভাগত ভগাব পাৰি।

১) চৰকাৰী বিধেয়ক :→

বিধায়িনী প্ৰক্ৰিয়াৰ বাবে সংসদত মূলত: দুই ধৰণৰ বিধেয়ক উত্থাপন কৰা হয়। সেয়ে চৰকাৰী বিধেয়ক আৰু বেচৰকাৰী বিধেয়ক। চৰকাৰী বিধেয়ক সংসদৰ যিকোনো সদনত উত্থাপন কৰিব পাৰি। এনে বিধেয়ক মন্ত্ৰীসকলে উত্থাপন কৰে। যিহেতু চৰকাৰৰ প্ৰতি সংসদত সংখ্যাগৰিষ্ঠৰ সমৰ্থন থাকে সেয়েহে এনে বিধেয়ক সদনত গৃহীত

কৰাৰ পাছতহে এই বিধেয়ক সমূহে আইনত পৰিণত হয়।

সংসদৰ এখন সদনত বিধেয়ক এখন গৃহীত হোৱাৰ পিছত আনখন সদনলৈ অনুমোদনৰ বাবে পঠিওৱা হয়। আনখন সদনতো বিধেয়কখন বিভিন্ন পৰ্যায়ৰ মাজেৰে আলোচনা কৰা হয়। সেয়েত আনখন সদনে বিধেয়কখন —

(i) কোনো সংশোধন নকৰাকৈ বিধেয়কখন প্ৰথম সদনে দিয়া ধৰণে গ্ৰহণ কৰিব পাৰে,

(ii) বিধেয়কখন সম্পূৰ্ণভাৱে নাকচ কৰিব পাৰে,

(iii) বিধেয়কখন সংশোধনৰ নিৰ্দেশ কৰি পুনৰ বিবেচনাৰ বাবে প্ৰথম সদনলৈ ঘূৰাই পঠিয়াব পাৰে।

(iv) কোনো সিদ্ধান্ত নোলোৱাকৈ সম্পূৰ্ণ নিষ্ক্ৰিয়ভাৱে ফেলাই ৰাখিব পাৰে।

(4) <u>সামৰণি</u> :-

আগতেই উল্লেখ কৰাৰ দৰে ভাৰতীয়

সাংবাদৰ উভয় সদনত বিল বাৰ্য কৰা হয়। কিন্তু সাংবিধানে ৰাজ্যসভাৰ তুলনাত লোকসভাৰ হাতত যথেষ্ট ক্ষমতা অৰ্পন কৰিছে। যদিও ৰাজ্যসভাৰ অবদান নুই কৰিব নোৱাৰি।

ভাৰতৰ দৰে এখন গণতান্ত্ৰিক দেশত বিল বা বিধেয়ক এক উল্লেখনীয় বিষয়। এই বিল বিভিন্ন প্ৰক্ৰিয়াৰে মাজেৰে অহিনলৈ পৰিণত হয়, দেশ সুকলমে চলাবলৈ যথেষ্ট সহায় কৰে।

(৬) প্ৰসংগ পুথি :→

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ছবুৰ ড° আব্দুচ্

ৰায়, সুদৰ্শন, ২০২০ সংসদীয় কাৰ্য আৰু দও দেৱজিৎ অনুশীলন ইউনিয়ন বুক পাব্লিকেচন।

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Certificate

This is to certify that Miss Trishna Kalita has worked under my Guidance and supervision for her project work on SEC COURSE, which is being submitted to the Dept. of Political Science.

Date : 28/05/2023

Place : Nagarbera

(Dr. Diganta Kalita)
Asst. Professor

বিষয়সূচী

(Content)

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~~প্ৰশ্ন~~ 1. ভাৰতবৰ্ষত বিধেয়ক গৃহিত কৰণ প্ৰক্ৰিয়াটো আলোচনা কৰা ?

~~উত্তৰ~~ :- ভাৰতবৰ্ষত বিধেয়ক গৃহিত কৰণ প্ৰক্ৰিয়াটো তলত আলোচনা কৰা হ'ল —

আৰম্ভণি :- কেন্দ্ৰীয় বিধানমণ্ডল বা সংসদৰ আটাইতকৈ গুৰুত্বপূৰ্ণ কাৰ্যটোৱেই হৈছে দেশৰ প্ৰশাসন কাৰ্য পৰিচালনা কৰাৰ ক্ষেত্ৰত প্ৰয়োজন হোৱা আইন সমূহ প্ৰণয়ন কৰা। যাৰ আধাৰত দেশৰ কাৰ্য-পালিকাই প্ৰশাসনকাৰ্য সুচাৰুৰূপে পৰিচালনা কৰাৰ প্ৰয়াস কৰে। এই প্ৰশাসনকাৰ্য সু-শৃংখলাৰে পৰিচালনা কৰিবৰ বাবে প্ৰয়োজন হোৱা এই আইনসমূহক আৰম্ভণিতে এক খচৰাবিধি অথবা প্ৰস্তাৱ আকাৰে অনুমোদনৰ বাবে সংসদত উত্থাপন কৰা হয়। অৰ্থাৎ আইনৰ এই খচৰাবিধি বা প্ৰস্তাৱসমূহকে সাধাৰণতে বিল বা বিধেয়ক হিচাপে বিবেচনা কৰা হয়। এই বিল বা বিধেয়কত

নিৰ্ধাৰিত প্ৰক্ৰিয়াত আজেৰে সংসদে অনুমোদন জনালে আৰু ৰাষ্ট্ৰপতিয়ে যদি তাত সন্মতি প্ৰদান কৰে তেন্তে ই আইনলৈ ৰূপান্তৰিত হয়। এইবোৰ হৈছে কিছুমান বৈধানিক প্ৰস্তাৱ, যিবোৰক বিল অথবা বিধেয়কৰ ৰূপত সংসদৰ মজিয়াত উত্থাপন কৰা হয়। অৱশেষত এই আইনসমূহক ভাৰতীয় সংবিধানত উল্লেখ কৰা হয়। আইন প্ৰণয়নৰ বাবে সংসদত উত্থাপন কৰা এই বিল বা বিধেয়ক সমূহ বিভিন্ন প্ৰকাৰৰ হ'ব পাৰে।

বিধেয়কৰ পৰিচয় :- সংসদৰ অনুমোদনৰ কাৰণে আইনৰ যি প্ৰস্তাৱ উত্থাপন কৰা হয় তাক বিধেয়-ক বা বিল বোলে। গতিকে বিধেয়ক হ'ল সংসদত প্ৰস্তাৱিত আইন এখনৰ খচৰা। অৰ্থাৎ আইনৰ পাণ্ডুলিপিক বিধেয়ক বা বিল বোলা হয়। বিধেয়ক এখন সাংবিধানিক প্ৰক্ৰিয়া অনুসৰি গৃহীত হোৱাৰ পিছতহে আইনত পৰিণত হয়।

আধাৰণতে সংসদত উত্থাপন কৰা বিধেয়কক দুই ভাগত বিভক্ত কৰা হয়। যেনে - ক) চৰকাৰী

বিধেয়ক আৰু খ) বেচৰকাৰী বিধেয়ক ।

সেইদৰে চৰকাৰী বিধেয়কক আকৌ — (i) সাধাৰণ বিধেয়ক, অৰ্থ সংক্ৰান্ত বিধেয়ক আৰু বিত্ত সংক্ৰান্ত বিধেয়ক আদি ভাগত ভাগ কৰা হয় । এইখিনিতে এটা কথা উল্লেখযোগ্য যে বহুক্ষেত্ৰত অৰ্থ সংক্ৰান্ত বিধেয়ক আৰু বিত্ত সংক্ৰান্ত বিধেয়কক একেটা অৰ্থত ব্যৱহাৰ কৰা দেখা যায় । কিন্তু প্ৰকৃতপক্ষে এই দুয়োটা বিধেয়কৰ মাজত কিছুমান পাৰ্থক্য আছে ।

অৰ্থ সংক্ৰান্ত বিধেয়ক আৰু বিত্ত বিধেয়কৰ মাজৰ পাৰ্থক্যসমূহ হ'ল —

i) অৰ্থ সংক্ৰান্ত বিধেয়ক সম্পৰ্কে সংবিধানৰ ১১০ নং অনুচ্ছেদত আলোচনা কৰা হৈছে । আনহাতে বিত্ত বিধেয়ক সম্পৰ্কে সংবিধানৰ ১১৭ নং অনুচ্ছেদত আলোচনা কৰা হৈছে ।

ii) অৰ্থ সংক্ৰান্ত বিধেয়ক উত্থাপনৰ ক্ষেত্ৰত ৰাষ্ট্ৰপতিৰ আগতীয়া অনুমোদনৰ প্ৰয়োজন হয় । কিন্তু বিত্ত বিধেয়ক উত্থাপনৰ ক্ষেত্ৰত এনে ধৰণৰ

অনুমতিৰ প্ৰয়োজন নহয়।

(iii) অৰ্থ সংক্ৰান্ত বিধেয়কৰ ক্ষেত্ৰত লোকসভাৰ অধ্যক্ষই প্ৰমাণ পত্ৰ দিব লাগে। কিন্তু বিত্ত বিধেয়কৰ ক্ষেত্ৰত এনে প্ৰমাণ পত্ৰৰ প্ৰয়োজন নহয়।

(iv) অৰ্থ সংক্ৰান্ত বিধেয়ক অকল মাত্ৰ লোকসভাতহে উত্থাপন কৰিব পাৰি। কিন্তু বিত্ত বিধেয়ক লোকসভা বা ৰাজ্যসভাত উত্থাপন কৰিব পাৰি।

এইদৰে দেখা যায় যে অৰ্থ সংক্ৰান্ত বিধেয়ক আৰু বিত্ত বিধেয়কৰ মাজত ভালেমান পাৰ্থক্য আছে। এই পাৰ্থক্যসমূহ অৱগত কৰা আৰু ছাত্ৰ-ছাত্ৰীসকলক বিধেয়কৰ প্ৰকাৰ সম্পৰ্কে স্পষ্ট আভাস দিয়াৰ কাৰণে এই অধ্যায়ত প্ৰয়াস কৰা হৈছে।

এই প্ৰসংগতে আমি চৰকাৰী আৰু

বিবেচনাৰ বাবে তেওঁৰ বানীৰ সৈতে অংসদলৈ ওভোতাই পঠিয়াব পাৰে। এনে ক্ষেত্ৰত অংসদে পুনৰ্বিবেচনা কৰি বিধেয়কখন আকৌ ৰাষ্ট্ৰপতিলৈ প্ৰেৰণ কৰিলে ৰাষ্ট্ৰপতিয়ে তাত সন্মতি প্ৰদান কৰিবলৈ বাধ্য। কোনো বিধেয়কত ৰাষ্ট্ৰপতিয়ে স্বাক্ষৰ দান কৰাৰ পিছত ই আইনত পৰিণত হয় আৰু চৰকাৰে নিৰ্দ্ধাৰণ কৰা তাৰিখৰ পৰা সেই আইন বলবৎ কৰা হয়। আইনত পৰিণত হোৱাৰ পাছত ইয়াক চৰকাৰী ৰাজ-পত্ৰত প্ৰকাশ কৰা হয়।

আমবনি :— মিলেই নথকা, অন্যান্য আৰ্থিক বিধেয়ক বা চৰকাৰী বিধেয়কৰ দৰে একেই প্ৰক্ৰিয়া আৰু নীতি নিয়মৰ মাজেদি অংসদৰ দুয়ো সদনে বেচৰকাৰী বিধেয়ক গৃহীত কৰাৰ

পিছত ইয়াক ৰাষ্ট্ৰপতিৰ অনুমতিৰ কাৰণে প্ৰেৰণ কৰা হয়। ৰাষ্ট্ৰপতিৰ অনুমতিৰ পিছত ই আইনত পৰিণত হয় আৰু নিৰ্দিষ্ট সময়ৰ পৰা কাৰ্যকৰী হয়।

এইদৰে ভাৰতীয় সংবিধানে ১০৭ নং অনুচ্ছেদৰ পৰা ১২২ নং অনুচ্ছেদলৈ বিধায়িনী প্ৰক্ৰিয়াৰ সাংবিধানিক ব্যৱস্থা, বিধেয়কৰ প্ৰকাৰ ইত্যাদি বিষয়বস্তুবোৰ আলোচনা কৰিছে। সংবিধানত বিধায়িনী ব্যৱস্থাৰ দীঘলীয়া আলোচনা কৰাৰ উদ্দেশ্য হৈছে বিধি বিধান সম্পৰ্কত স্পষ্ট ধাৰণা তৈয়াৰ কৰা। এই ক্ষেত্ৰত আমাৰ সংবিধান নিৰ্মাতাসকল সফল হৈছে বুলি ক'ব পাৰি।

প্ৰসংগ পুথি :- উক্ত কথাখিনি ড° সুবুৰ.

P.T.O

আব্দুছ , ৰায় সুদৰ্শন , ড° দত্ত দেৱজিৎ এই কেইগৰাকী লেখকৰ "সংসদীয় কাৰ্য - প্ৰণালী আৰু অনুশীলন" নামৰ কিতাপ খনৰ পৰা লিখা হৈছে ।

পৃষ্ঠা নং :- 22 - 30 ; 41 - 46

2

3

1 প্ৰকল্প (PROJECT)

SEC COURSE

শিৰোনাম : ভাৰতবৰ্ষত বিত্তীয়ক ইন্ধিতকৰণ প্ৰক্ৰিয়া ।

জমা কৰ্তাৰ নাম : লিপিকা বৰ্মন

শ্ৰেণী : তৃতীয় ষাণ্মাসিক

ৰোল নং : 163

2.

Certificate

This is to certify that

miss Lipika Barman has worked under my Guidance and Supervision for her project work on SEC course, which is being submitted to the Dept of Political Science.

Date : 28/05/2023
place : Nagarbera

(Dr. Diganta Kalita)
Asst. professor

বিষয় সূচী - (content)   পৃষ্ঠা নং -

বিধেয়ক গৃহীতকৰণ প্ৰক্ৰিয়া —

~~উত্তৰঃ~~

① পাতনি :-

সংবিধান হৈছে কোনো এখন দেশৰ শাসন ব্যৱস্থাৰ বৈধ দলিল আৰু চূড়ান্ত আইন। ইয়াৰ জৰিয়তে দেশৰ শাসন ব্যৱস্থা, বিধায়িনী ব্যৱস্থা আৰু ন্যায় ব্যৱস্থা, আদিৰ লগত জড়িত বিষয়বোৰ সুসমঞ্জস্যৰে পৰিচালনা কৰা হয়। সংবিধানক ভিত্তি কৰি গঢ়ি উঠে কোনো এখন দেশৰ শাসন ব্যৱস্থা আৰু জাতীয় পৰিচয়। দেশৰ সভ্যতা, সংস্কৃতি, মানুহৰ চৰিত্ৰ, পৰম্পৰা, ৰীতি নীতি, জীৱন ধাৰণ পদ্ধতি ইত্যাদি সকলো ক্ষেত্ৰতে সংবিধানৰ প্ৰত্যক্ষ অথবা পৰোক্ষ প্ৰভাৱ পৰা দেখিবলৈ পোৱা যায়। সেয়েহে সংবিধানক এখন দেশৰ শাসন ব্যৱস্থাৰ আত্মা বুলি কোৱা হয়। ভাৰতীয় সংবিধানৰ আটাইতকৈ গুৰুত্বপূৰ্ণ কামবোৰৰেই আইন প্ৰণয়ন কৰা। সংবিধিনে ভাৰতীয় আইন প্ৰণয়নৰ দায়িত্ব কেন্দ্ৰীয় আইন সভা আৰু ৰাজ্যিক আইন সভাৰ হাতত অৰ্পণ কৰিছে। কেন্দ্ৰীয় সংসদে কেন্দ্ৰীয় তালিকা ভুক্ত বিষয় সূচী আৰু প্ৰয়োজনবোধে ৰাজ্যিক তালিকা ভুক্ত বিষয়সূচীৰ ওপৰতো আইন

প্ৰণয়ন কৰিব পাৰে।

② বিল সম্পৰ্কীয় পৰিচয়:—

আইন প্ৰণয়নৰ উদ্দেশ্যে সংসদে যিবোৰ প্ৰস্তাৱ দাঙি ধৰা হয়। সেই প্ৰত্যেকটো প্ৰস্তাৱকে বিল আকাৰে উপস্থাপন কৰা হয়। সংসদে উত্থাপিত আইনৰ খচৰা বা প্ৰস্তাৱকে বিল বুলি কোৱা হয়।

বাংলাদেশৰ সংবিধানৰ ৮০ নং অনুচ্ছেদত বিল সম্পৰ্কে উল্লেখ আছে।

সংসদৰ অনুমোদনৰ কাৰণে আইনৰ যি প্ৰস্তাৱ উত্থাপন কৰা হয় তাক বিধেয়ক বোলে। গতিকে বিধেয়ক হ'ল সংসদত প্ৰস্তাৱিত আইন এখনৰ খচৰা অৰ্থাৎ আইন পাণ্ডুলিপি তাক বিধেয়ক বা বিল বোলা হয়। বিল এখনক সাংবিধানিক প্ৰক্ৰিয়া অনুসৰি গৃহীত হোৱাৰ পিছতহে আইনত পৰিণত হয়। এখন বিধেয়ক এটা নিৰ্দিষ্ট শিৰোনামেৰে আৰম্ভ হয়। বিধেয়ক এখনত এটি প্ৰস্তাৱনা আৰু কাৰ্যকৰণৰ সূত্ৰ থাকে। তদুপৰি বিধেয়ক এখনৰ প্ৰয়োগ যোগ্যতা পৰিমাণ দিয়া হয় আৰু কিছু ক্ষেত্ৰত ই আইনত পৰিণত হোৱাৰ তাৰিখো।

নিৰ্দিষ্টকৈ দিয়া হয়।

বিল বা বিধেয়ক নিৰ্ধাৰিত প্ৰক্ৰিয়াৰে সংসদে অনুমোদন জনালে আৰু ৰাষ্ট্ৰপতিয়ে যদি তাত সন্মতি প্ৰদান কৰে তেন্তে ই আইনলৈ ৰূপান্তৰিত হয়। এইয়াৰ হৈছে কিছুমান বৈধানিক প্ৰস্তাৱ. মিথিলাক বিল অথবা বিধেয়কৰ ৰূপত সংসদৰ অভিমতে উত্থাপন কৰা হয়।

বিধেয়কৰ প্ৰকাৰ ঃ

দেশৰ কেন্দ্ৰীয় সংসদ হৈছে আইন প্ৰণয়নৰ সৰ্বোচ্চ সংস্থা (Highest Law making organ) সংসদৰ উচ্চ সদন আৰু নিম্নসদনে যথাক্ৰমে ৰাজ্য সভা আৰু লোক সভাৰ প্ৰধান কাৰ্য হৈছে দেশৰ বাবে প্ৰয়োজনীয় আইন ৰচনা কৰা. পুৰণি আইন সংশোধন বা পৰিৱৰ্তন কৰি দেশৰ শাসন ব্যৱস্থা শৃংখলিত কৰা। কেন্দ্ৰীয় সংসদে প্ৰণয়ন কৰা বিধেয়কত ৰাষ্ট্ৰপতিৰ স্বাক্ষৰৰ প্ৰয়োজন হয়। অৰ্থাৎ ৰাষ্ট্ৰপতিৰ স্বাক্ষৰৰ পিছতহে কোনো বিধেয়ক আইনত পৰিণত হয়।

সাধাৰণতে সংসদত উত্থাপন কৰা বিধেয়কক দুই ভাগত বিভক্ত কৰা হয়।

সেই দুটা হ'ল—

① চৰকাৰী বিধেয়ক আৰু

② বেচৰকাৰী বিধেয়ক।

সেইদৰে চৰকাৰী বিধেয়কক আকৌ সাধাৰণ বিধেয়ক

কোনো বিষয়ত চৰকাৰৰ দৃষ্টি আকৰ্ষণ কৰা।

⑦ এই বিধেয়ক ৰাজ্যসভা বা লোকসভাত উত্থাপন কৰিব পাৰি।

⑧ সাধাৰণতে সংসদৰ বিৰোধী পক্ষৰ সদস্যই বেচৰকাৰী বিধেয়ক উত্থাপন কৰা দেখা যায়।

⑥ সামৰণি:

ওপৰত উল্লেখ কৰি অহা মতে ভাৰতীয় সংবিধান মতে গঠন হোৱা প্ৰথম খন কেন্দ্ৰীয় চৰকাৰৰ পৰা বৰ্তমানৰ কেন্দ্ৰীয় চৰকাৰৰ সময়লৈ প্ৰায় 300খন বেচৰকাৰী বিধেয়ক সংসদত উত্থাপন হোৱা দেখা যায়। সেই 300 খন বিধেয়কৰ মাজৰ পৰা মাত্ৰ ১৪ খন বেচৰকাৰী বিধেয়ক সংসদৰ দ্বাৰা গৃহিত হৈ আইনৰ মৰ্যদা লাভ কৰিছে। সংসদৰ এই তথ্যই দেখুৱায় যে সংসদত মাত্ৰ ৪ শতাংশ বেচৰকাৰী বিধেয়ক গৃহীত হোৱাৰ বিপৰীতে ৯৬ শতাংশ বেচৰকাৰী বিধেয়ক নাকচ হৈছে বা সেইবোৰৰ ম্যাদ উকলি গৈছে।

যিয়েই নহওক কিয়, অন্যান্য সাধাৰণ বিধেয়ক বা চৰকাৰী বিধেয়কৰ দৰে একেই প্ৰক্ৰিয়া আৰু নীতি নিয়মৰ মাজেদি সংসদৰ উভয় সদনে বেচৰকাৰী বিধেয়ক গৃহীত কৰাৰ পিছত ইয়াক ৰাষ্ট্ৰপতিৰ সন্মতিৰ কাৰণে প্ৰেৰণ কৰা হয়। ৰাষ্ট্ৰপতিৰ সন্মতিৰ -

পিছত ই আইন পৰিণত হয় আৰু নিৰ্দিষ্ট সময়ৰ পৰা কাৰ্যকৰী হয়।

প্ৰসংগ পুথি :

ড° ফণীন্দ্ৰ কলিতা
দিবাকৰ দাস
২০২০: সংসদীয় প্ৰক্ৰিয়া আৰু কাৰ্য পদ্ধতি সমূহ।
পৃষ্ঠা → 26 – 33

ড° আব্দুছ ছবুৰ
সুদৰ্শন ৰায়
ড° দেৱজিৎ দত্ত
২০২০: সংসদীয় কাৰ্যপ্ৰণালী আৰু অনুশীলন। ইউনিয়ন বুক পাব্লিকেচন।
পৃষ্ঠা 29, 30