2016

STATISTICS

(Major)

Paper : 1.1

(Descriptive Statistics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed (reasoning is not necessary) : 1×7=7

(a) State the definition of mean deviation in words.

(b) If x_i / f_i $(i = 1, 2, ..., n)$ is a frequency distribution and $u_i = \frac{x_i - a}{h}$, then which one of the following is true?

(i) $\bar{x} = h\bar{u}$

(ii) $\bar{x} = a + h\bar{u}$

(iii) $\bar{x} = a + \bar{u}$

(iv) $\bar{x} = a - h\bar{u}$

[Symbols have their usual meanings.]

(c) The standard deviations of the following two series

Series–1 : 1, 2, 3, 4
Series–2 : 3, 4, 5, 6

are

(i) same

(ii) not the same

(iii) sometimes same and sometimes not

(iv) None of the above

[You have to answer without doing any calculation.]

(d) The geometric mean of $3, 3^2, ..., 3^n$ is

(i) $3^{n/2}$

(ii) $3^{\frac{n+1}{2}}$

(iii) 3^n

(iv) $3^{\frac{n(n+1)}{2}}$

(e) State whether the following statement is true :

"Coefficient of variation is invariant of unit of measurement."

(f) Mention any one use of range.

(g) If $\hat{x} = 3$ and

$$A = \sum_{i=1}^{n}(x_i - 3)^2,\ B = \sum_{i=1}^{n}(x_i - 2)^2,\ C = \sum_{i=1}^{n}(x_i - 4)^2$$

then minimum of A, B, C is

(i) B

(ii) C

(iii) A

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) Write the expressions for second and fourth central moments in terms of raw moments. 1+1=2

(b) Write any two measures of kurtosis.

(c) Suppose you have the following :

x :	2	4	6	8
y :	4	6	8	10

Then which one of the following is correct?

(i) $b_{yx} = 1$

(ii) $b_{yx} = -1$

(iii) $b_{yx} = 0$

(iv) $b_{yx} = \infty$

[Give reason to support your answer.]

(d) Define the Karl-Pearson correlation coefficient. Also state what does it measure. 1+1=2

3. Answer any *three* of the following : 5×3=15

(a) Define mode and derive its formula.

(b) Define arithmetic mean (AM) and harmonic mean (HM). Show also that AM ≥ HM. [You have to prove it for n values.] 2+3=5

(c) Show that the correlation coefficient lies between −1 and +1. Also give the geometrical interpretation of the case when $r = +1$. 4+1=5

(d) If a variable takes values 0, 1, 2, 3, ⋯ with frequencies proportional to $e^{-\lambda}$, $\lambda e^{-\lambda}, \frac{e^{-\lambda}\lambda^2}{2!}, \frac{e^{-\lambda}\lambda^3}{3!}$, then find the mean and variance of the distribution.

(e) For any discrete series, show that mean deviation about mean is less than standard deviation.

4. **EITHER**

(a) Compare and contrast amongst different measures of dispersion. 5

(b) Establish the relationship between standard deviation and range. 5

OR

5. (a) For two variables X and Y, find the line of regression of Y on X. 5

(b) Show that the mean deviation is least when measured about median. 5

6. **EITHER**

Define cumulants. Establish its additive property. With usual notation show that $\mu_2 = \kappa_2$. 3+4+3=10

OR

7. (a) Suppose you want to fit a third degree polynomial of the type

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

to the set of points (x_i, y_i), $i = 1, 2, \cdots, n$, where a_0, a_1, a_2 and a_3 are constants. Derive the normal equations to obtain the constants a_0, a_1, a_2 and a_3. 5

(b) Explain how would you fit the polynomials of the following types : 2½×2=5

(i) $y = ax^b$

(ii) $y = ab^x$

8. **EITHER**

(a) Find the arithmetic mean and variance of the observations $1 \cdot 3 \cdot 5, 3 \cdot 5 \cdot 7, 5 \cdot 7 \cdot 9 \ldots (2n-1)(2n+1)(2n+3)$. 2+3=5

(b) Write a note on skewness in detail. 5

OR

9. Write explanatory notes on any *one* of the following : 10

(a) Orthogonal polynomials

(b) Correlation ratio

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