Total No. of printed pages = 5

3 (Sem 3) STS M1

2015

STATISTICS

(Major)

Paper : 3.1

(Mathematical Methods-II)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer all parts of the questions : $1\times7=7$

(a) When is a matrix said to be in normal form ?

(b) State the condition that a matrix A has to satisfy to be an orthogonal matrix.

(c) Find the rank of the following matrix :

$$A=\begin{pmatrix}1 & 3 & 4\\ 1 & 2 & 6\end{pmatrix}$$

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(d) Justify that the matrices

$$A=\begin{pmatrix}1 & 3 & 4 & 5\\ 1 & 2 & 6 & 7\\ 1 & 5 & 0 & 1\end{pmatrix}, \quad B=\begin{pmatrix}1 & 3 & 4 & 5 & 12\\ 1 & 2 & 6 & 7 & 18\\ 1 & 5 & 0 & 1 & 0\end{pmatrix}$$

have the same rank.

(e) Given that for a square matrix A $\det(A) = 18$, find $\det(A^T)$.

(f) Define a positive definite matrix.

(g) Given $\rho(A) = 3$, where $\rho(A)$ denotes the rank of A. What is $\rho(A^T)$?

2. Answer any *three* parts of the questions : 5×3=15

(a) Prove that for any two matrices $(A)_{m\times n}$ and $(B)_{n\times q}$

$$\rho(AB) \le \min(\rho(A), \rho(B))$$

where $\rho(A)$ denotes rank of matrix A.

(b) Show that if A is an orthogonal matrix then A^T and A^{-1} are also orthogonal.

(c) If A be a n×n matrix, prove that

$$|\text{adj}\,A| = |A|^{n-1}$$

(d) Prove that the determinant of an orthogonal matrix is either +1 or −1.

(e) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| \neq 0$.

3. Answer any *three* parts of the questions : 10×3=30

(a) Find a solution to the system of linear equations given by

$$2x + 7z = 4$$
$$3x + 3y + 6z = 3$$
$$2x + 2y + 4z = 2$$

(b) Find the inverse of the matrix

$$S=\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}$$

and show that SAS^{-1} is a diagonal matrix where

$$A=\frac{1}{2}\begin{pmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{pmatrix}$$

(c) Compute the inverse of the following matrix by using elementary transformations

$$\begin{pmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{pmatrix}$$

(d) Examine if the quadratic form

$$6x^2+17y^2+3z^2-20xy-14yz+8zx$$

is positive semi definite.

(e) Let A be a matrix of order $m\times n$ and let rank (A) = r. Then show that there exist two non singular square matrix P and Q (where P is of order $m\times m$ and Q is of order $m\times n$) such that

$$PAQ=\begin{bmatrix} I_{r\times r} & O_{r\times\overline{n-r}} \\ O_{\overline{m-r}\times r} & O_{\overline{m-r}\times\overline{n-r}} \end{bmatrix}$$

4. Answer all parts of the questions. $2\times4=8$

(a) State all the rules of elementary transformation.

(b) Let A be a 3×4 matrix. Giving reasons, examine if rank (A) can be 4 or higher.

(c) Suppose we have the system of equations $AX = 0$, where there are m equations in n unknown. Also rank (A) = r. State the nature of solution.

(d) Find the rank of the matrix

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$