2017

MATHEMATICS

(Major)

Paper : 5.6

(Optimization Theory)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Fill in the blanks : 1×7=7

(a) Two hyperplanes are said to be parallel, if they have the same unit _____.

(b) Convex hull of a set $A \subseteq \mathbb{R}^n$ is the smallest _____ set containing A.

(c) If the dual problem is unbounded, then the primal problem does not have any _____ solution.

(d) In the canonical form of an LPP, the objective function is of the _____ type.

(e) If the k-th constraint of the primal is an equation, then the corresponding dual variable is _____ in sign.

(f) In a balanced transportation problem with m origins and n destinations, there are atmost _____ basic variables.

(g) A system of m linear equations in n unknowns has atmost _____ basic solutions.

2. Answer the following questions : 2×4=8

(a) Define a convex cone.

(b) Show that the hyperplane

$$H = \{x \in \mathbb{R}^n : Cx = z\}$$

is a convex set.

(c) Determine the convex hull of the set

$$A = \{x = (x_1, x_2) : x_1^2 + x_2^2 = 1\}$$

(d) Reduce the following LPP to its standard form :

$$\text{Maximize} \quad Z = x_1 - 3x_2$$

subject to

$$-x_1 + 2x_2 \le -15$$
$$x_1 + 3x_2 \ge 10$$
$$x_1, x_2 \ge 0$$

3. Answer any *three* parts of the following : 5×3=15

(a) Show that the set

$$A = \{(x_1, x_2) \in \mathbb{R}^2 : x_1, x_2 \le 1, x_1 x_2 \ge 0\}$$

is not a convex set.

(b) Obtain all the basic solutions to the following system of linear equations :

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

How many of the basic solutions are degenerate?

(c) Show that the set of all convex combinations of a finite number of points of a set $S \subseteq \mathbb{R}^n$ is a convex set.

(d) Use graphical method to show that there exists an alternative optima for the following LPP :

Maximize $Z = 2x_1 + 4x_2$

subject to the constraints

$$x_1 + 2x_2 \le 5$$
$$x_1 + x_2 \le 4$$
$$x_1,\ x_2 \ge 0$$

(e) Prove that the dual of the dual of a primal LPP is the primal itself.

4. Use simplex method to solve the following LPP : 10

Maximize $Z = 12x_1 + 3x_2 + x_3$

subject to

$$10x_1 + 2x_2 + x_3 \le 100$$
$$7x_1 + 3x_2 + 2x_3 \le 77$$
$$2x_1 + 4x_2 + x_3 \le 80$$
$$x_1,\ x_2,\ x_3 \ge 0$$

Or

Use Charne's M method to solve the following LPP :

Maximize $Z = x_1 + 5x_2$

subject to

$$3x_1 + 4x_2 \le 6$$
$$x_1 + 3x_2 \ge 3$$
$$x_1,\ x_2 \ge 0$$

5. Using two-phase method, show that a feasible solution to the following problem does not exist : 10

Maximize $Z = x_1 + 2x_2 + 3x_3$

subject to

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1,\ x_2,\ x_3 \ge 0$$

Or

Formulate the following problem as an LPP and also formulate its dual :

A person requires minimum 10, 12, 12 units of chemicals *A*, *B*, *C* respectively for his garden. A liquid product contains 3, 2, 1 units of *A*, *B*, C respectively per jar. A dry product contains 1, 2, 4 units of *A*, *B*, *C* respectively per packet. The person wants to make the investment for his garden as minimum as possible, where it is given that the liquid product sells at ₹ 2 per jar and the dry product sells at ₹ 1 per packet.

6. A company has three plants *A*, *B*, *C* and three warehouses *X*, *Y*, *Z*. The number of units available at the plants are 60, 70, 80 respectively. The demands at *X*, *Y*, *Z* are 50, 80, 80 respectively. The unit cost of transportation are given in the following table :

	X	*Y*	*Z*
A	1	7	3
B	3	8	9
C	11	3	5

Find the allocation so that the total transportation cost is minimum. 10

Or

A company has 5 jobs to be done. The following matrix shows the return in ₹ of assigning *i*-th machine ($i = 1, 2, \cdots, 5$) to the *j*-th job ($j = 1, 2, \cdots, 5$). Assign the five jobs to the five machines so as to maximize the total expected profit :

Machines \ *Jobs*	*1*	*2*	*3*	*4*	*5*
A	5	11	10	12	4
B	2	4	6	3	5
C	3	1	5	14	6
D	6	14	4	11	7
E	7	9	8	12	5

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