2017

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write under what condition one may have an infinite number of great circles through two given points.

(b) Write any two coordinate systems to locate the position of a heavenly body on the celestial sphere.

(c) Explain what is meant by rising and setting of stars.

(d) Define primary circle.

(e) Write down six elements of a spherical triangle. Write also the relation between four consecutive elements of a spherical triangle.

(f) State the third law of Kepler.

(g) What is parallatic ellipse? Write down the equation of parallatic ellipse.

2. Answer the following questions : 2×4=8

(a) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(b) When α, δ be given, obtain a formula to find λ, β, where the symbols have their usual meanings.

(c) If θ be the angle subtended at the earth by the sun and a stationary point of a planet's orbit and ϕ be the maximum elongation of the planet, prove that $2\cot\theta = \sec\frac{\phi}{2} + \operatorname{cosec}\frac{\phi}{2}$.

(d) Explain the term 'direct and retrograde motion'.

3. Answer any *three* parts : 5×3=15

(a) Deduce Kepler's laws from the Newton's law of gravitation.

(b) Prove that the velocity of a planet at any point of its path varies inversely as the perpendicular from the sun upon the tangent to the path at that point. Also if V_1 and V_2 be the linear velocities of the planet at perihelion and aphelion respectively, then prove that $V_1 : V_2 = 1+e : 1-e$, where e is the eccentricity of the path of the planet.

(c) Prove that the apparent path of a star on account of parallax is an ellipse.

(d) What is twilight? Obtain the condition for twilight to last all night.

(e) If H be the hour angle of a star of declination δ when its azimuth is A and H' when the azimuth is $(180°+A)$, show that

$$\tan\phi = \frac{\cos\frac{1}{2}(H'+H)}{\cos\frac{1}{2}(H'-H)}$$

4. In any spherical triangle ABC, prove that

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Deduction from cosine formula is not allowed. Also prove that

$$\frac{\sin(A+B)}{\sin C} = \frac{\cos a + \cos b}{1+\cos c} \qquad 6+4=10$$

5. What is solar eclipse? Mention different types of solar eclipse. Also discuss (with neat diagram) the commencement of solar eclipse. 1+2+7=10

6. On account of refraction, the circular disc of the sun appears to be an ellipse. Prove it. 10

Or

(a) Assuming uniform motion of the sun in longitude, prove that the correction to the time of transit of a star of RA α, due to annual parallax has its greatest magnitudes $365\frac{1}{4}/2\pi \tan^{-1}(\sec\varepsilon \tan\alpha)$ days differ a solstice, ε being the obliquity of the ecliptic.

(b) Prove that the parallax increases the apparent semi diameter of the moon in the ratio $\sin z' : \sin(z' - \psi)$, where z' is the apparent zenith distance of the moon's centre and ψ is the angle subtended at the moon by the observer and the earth's centre (earth being assumed spherical). 10

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