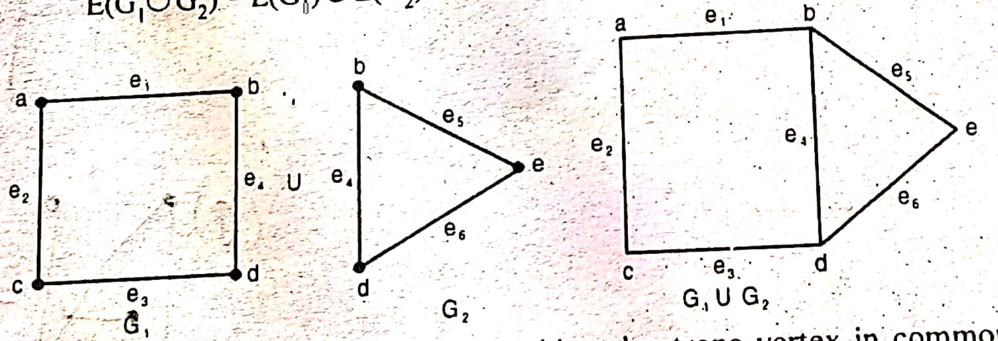
13.7. Operations of Graphs

In this section we would learn about some operations on the graph.

Union: Given two graphs G_1 and G_2 their union will be a graph such that

$$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$$

and
$$E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$$

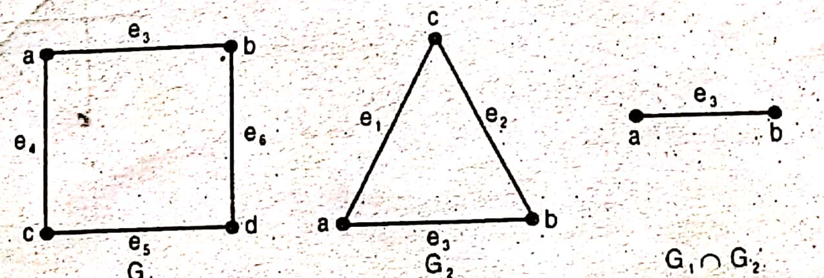


If G_1 & G_2 are regular graphs, then $G_1 \cup G_2$ may not be regular.

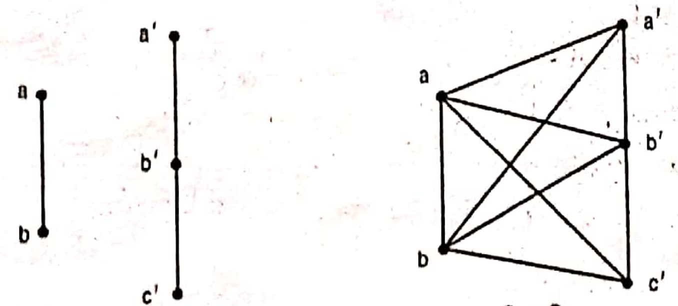
Intersection: Given two graphs G_1 and G_2 with at least one vertex in common then their intersection will be a graph such that

$$V(G_1 \cap G_2) = V(G_1) \cap V(G_2)$$

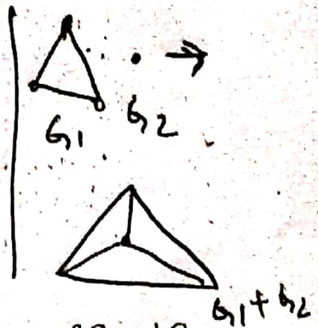
and
$$E(G_1 \cap G_2) = E(G_1) \cap E(G_2)$$



Sum of Two Graphs: If the graphs G_1 and G_2 such that $V_1(G_1) \cap V(G_2) = \phi$; then the sum $G_1 + G_2$ is defined as the graph whose vertex set is $V(G_1) + V(G_2)$ and the edge set is consisting those edges, which are in G_1 and in G_2 and the edges obtained, by joining each vertex of G_1 to each vertex of G_2. For example,



G_1 G_2 $G_1 + G_1$

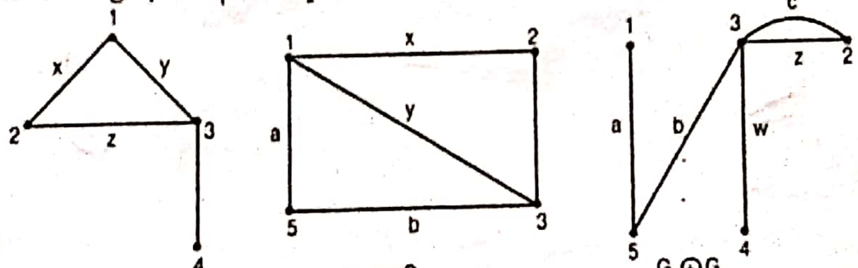


Ring sum: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs. Then the ring sum of G_1 and G_2, denoted by $G_1 \oplus G_2$, is defined as the graph G such that

(*i*) $V(G) = V(G_1) \cup V(G_2)$

(*ii*) $E(G) = E(G_1) \cup E(G_2) - E(G_1) \cap E(G_2)$ i.e., the edges that either in G_1 or G_2 but not in both.

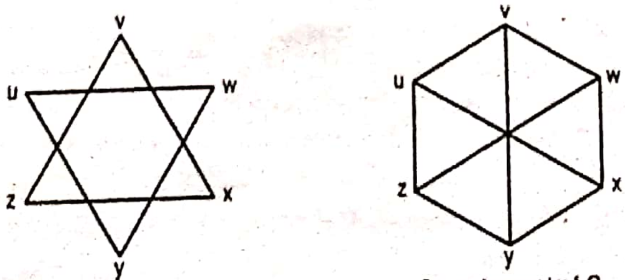
The Ring sum of two graphs G_1 and G_2 is shown below



G_1 G_1 $G_1 \oplus G_1$

Complement: The complement G' of G is defined as a simple graph with the same vertex set as G and where two vertices u and v are adjacent only when they are not adjacent in G.

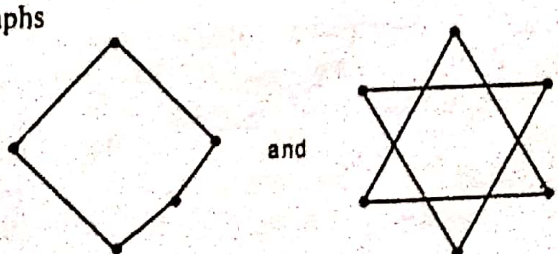
For example,



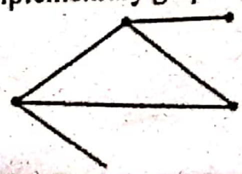
G Complement of G

A graph G is self-complementary if it is isomorphic to its complement.

For example, the graphs



self-complementary. The other self-complemenrary graph with five vertices is ,



Example 20. Show that every self-complementary graph has $4k$ or $4k+1$ vertices.

Solution: A graph with n vertices can be self-complementary only if the edge set of of k_n can be partitioned into two subsets of equal size.

Hence, the total no. of edges in k_n must be even. Now k_n has $\frac{n(n-1)}{2}$ edges which is even when $n = 4k$ or $4k+1$ and which is odd otherwise.

Product of Graphs : To define the product $G_1 \times G_2$ of two graphs consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever [$u_1 = v_1$ and u_2 and u_2 adj v_2] or [$u_2 = v_1$ and u_1 adj v_1]

For example,

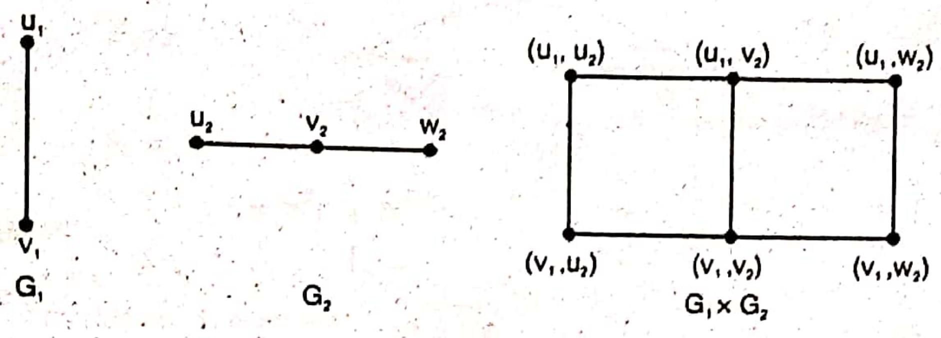


Fig. 13.25. The product of two graphs

Composition: The composition $G = G_1[G_2]$ also has $V = V_1 \times V_2$ as its point set, and $u = (u_1, u_2)$ is adjacent with $v = (v_1, v_2)$ whenever [u_1 adj v_1) or [$u_1 = v_1$ and u_2 adj v_1]. For the graphs G_1 and G_2 of Fig. 13.25 both compositions $G_1[G_2]$ and $G_2[G_1]$ are shown in Fig. 13.26.

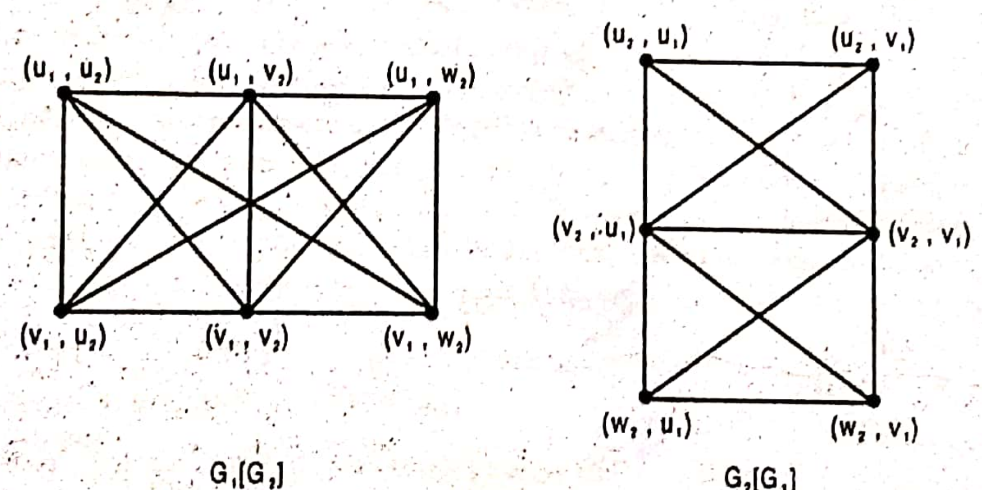


Fig. 13.26. Two compositions of graphs

Fusion: A pair of vertices v_1 and v_2 in graph G is said to be 'fused' if these two vertices are

Binary operations on graphs		
operation	No. of pts.	No. of lines
union $G_1 \cup G_2$	p_1+p_2	q_1+q_2
Join G_1+G_2	p_1+p_2	$q_1+q_2+p_1p_2$
Product $G_1 \times G_2$	p_1p_2	$p_1q_2+p_2q_1$
composition $G_1[G_2]$	p_1p_2	$p_1q_2+p_2^2q_1$

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Ex. Every self complementarey graph has $4n$ or $4n+1$ pts

Soln: Let $G(p,q)$ be a graph.

We know that the max^m value of q is $\frac{p(p-1)}{2}$

Again, given G is self complementory, then $G \cong \bar{G}$

$$\therefore X(G) + X(\bar{G}) = \frac{p(p-1)}{2}$$

Since G and $\bar{G}$ are isomorphic, so the no. of lines of G is equal to no. of lines of $\bar{G}$

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Unit 4

Connectivity and Line ...

of Menger's ...

Graph

(10) $\therefore |X(G)| + |X(\bar{G})| = \frac{p(p-1)}{2} \Rightarrow |X(G)| + |X(G)| = \frac{p(p-1)}{2}$

$\Rightarrow |X(G)| = \frac{p(p-1)}{4}$

since $|X(G)|$ is an integer, so we must have

either $p = 4n$ or $p-1 = 4n$

$\Rightarrow p = 4n+1$.

Q. At a bridge table
four different

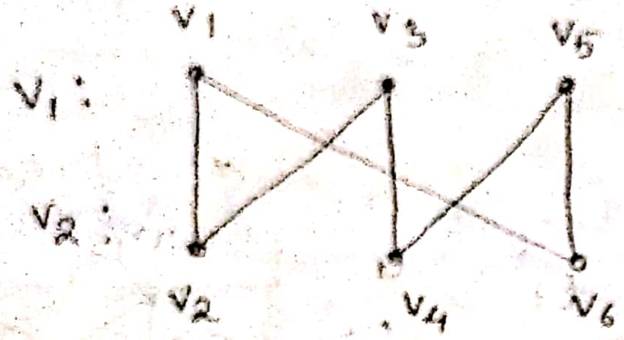
Thm: A graph is bipartite iff all its cycles (if exists) are even (ie G does not contain odd cycles)

Pf: If G is a bipartite graph, then its point set V can be partitioned into subsets V_1 and V_2 (say).

Every line of G joints a point V_1 to a point of V_2.

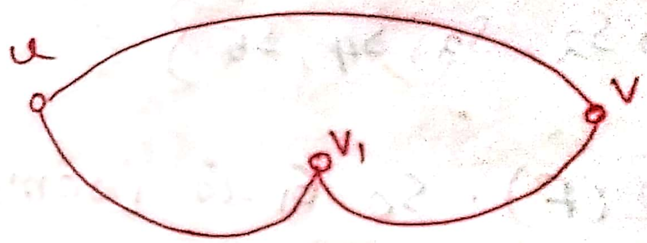
Thus every ~~of~~ cycle of length n say $v_1, v_2, \ldots v_{n-1} v_n$

in G necessarily has its oddly subscripted points in V_1 (say) and the others in V_2 so that its the length n is even.



Conversely, suppose we assume WLOG, that G is connected (otherwise we can consider the components of G separately). Let us take any point $v_1 \in V$. Let V_1 consists of v_1 and all points at even distance from v_1, while $V_2 = V - V_1$. Since all the cycles of G are even, every line of G joins a point of V_1 with a point of V_2. For suppose there is a line uv joining a points u and v of V_1. Then the union of geodesics from v_1 to v and from v_1 to u together with the line uv contains an odd cycle in G, a contradiction. H.W.

27



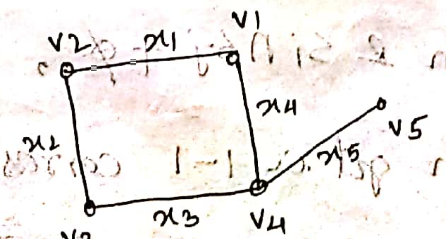
Def^n 08

Intersection graph: Let S be a set and $F = \{S_1, S_2, \ldots S_p\}$, a nonempty family of distinct non empty subsets of S whose union is S i.e. $\bigcup_{i=1}^{p} S_i = S$.

The intersection graph of F is denoted by $\Omega(F)$ and defined by $V(\Omega(F)) = F$ with S_i and S_j adjacent whenever $i \neq j$, $S_i \cap S_j \neq \phi$.

A graph G is an intersection graph on S if $\exists$ a family F of subsets of S for which $G \cong \Omega(F)$.

eg.



$S = \{v_1, v_2, v_3, v_4, v_5, x_1, x_2, x_3, x_4, x_5\}$

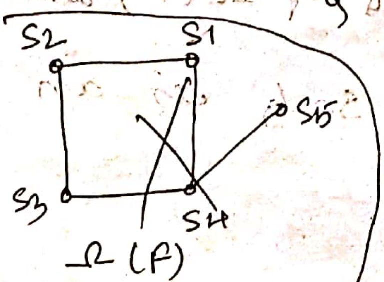
$S_1 = \{v_1, x_1, x_4\}$

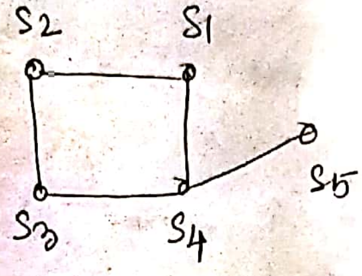
$S_2 = \{v_2, x_1, x_2\}$

$S_3 = \{v_3, x_2, x_3\}$

$S_4 = \{v_4, x_3, x_4, x_5\}$

$S_5 = \{v_5, x_5\}$





$\Omega(F)$

$F = \{S_1, S_2, S_3, S_4, S_5\}$

$G \cong \Omega(F)$. So G is isomorphic graph on the set S.

11.08.10
Wednesday

Thm: Every graph is an intersection graph.

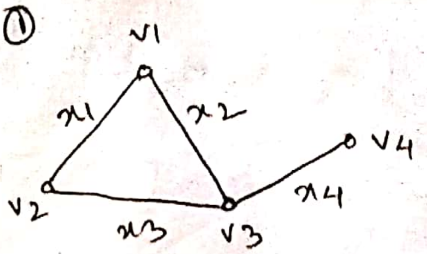
Pf: For each point v_i of G, let S_i denote the set containing the point v_i together with the lines incident with v_i.

$S_i = \{v_i\} \cup \{x_i$ if v_i incident with $x_i\}$

$S_j = \{v_j\} \cup \{x_j$ if v_j " " $x_j\}$

If v_i and v_j are adjacent in G, then S_i and S_j will have that line in common & $S_i \cap S_j \neq \phi$, so S_i adj S_j in $\Omega(F)$. So we can get a 1-1 correspondence between the points of G and the points of $\Omega(F)$ which preserves adjacency. Thus $G \cong \Omega(F)$. So G is an intersection graph.

eg. ①



$S = \{\overset{v_1}{x_1}, \overset{v_2}{x_2}, \overset{v_3}{x_3}, \overset{v_4}{x_4}\}$

$S_1 = \{v_1, x_1, x_2\}$

$S_2 = \{\cancel{x_2}\}\ \{v_2, x_1, x_3\}.$

$S_3 = \{v_3, x_2, x_3, x_4\}$

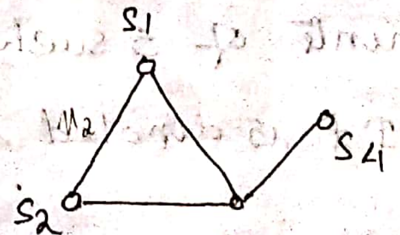
$S_4 = \{v_4, x_4\}$

$v_1 \to S_1$

$v_2 \to S_2$

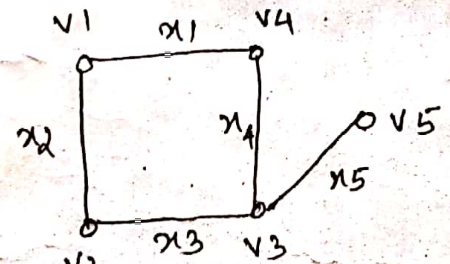
$v_3 \to S_3$

$v_4 \to S_4$



$\Omega(F)$.

②



$S = \{\overset{v_1}{x_1}, \overset{v_2}{x_2}, \overset{v_3}{x_3}, \overset{v_4}{x_4}, \overset{v_5}{x_5}\}$

$S_1 = \{v_1, x_1, x_2\}$

$S_2 = \{v_2, x_2, x_3\}$

$S_3 = \{v_3, x_3, x_4, x_5\}$

$S_4 = \{v_4, x_1, x_4\}$

$S_5 = \{v_5, x_5\}.$

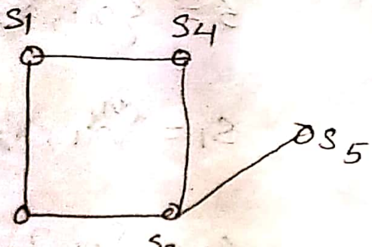
$V_1 \rightarrow S_1$

$V_2 \rightarrow S_2$

$V_3 \rightarrow S_3$

$V_4 \rightarrow S_4$

$V_5 \rightarrow S_5$



$\Omega(F)$.

<u>Intersection number</u> of a given graph G is the <u>minimum number</u> of elements of S such that G is an intersection graph on S. It is denoted by $\omega(G)$.