Liquid drop model

The liquid drop model of the nucleus as proposed by N. Bohr in 1937 uses the concept of potential barrier developed by Gramow.

According to this model, the nuclei of all elements behave like a liquid drop of electrically charged, incompressible liquid of constant density but of varying mass. The 'liquid' in reality consists of neutron and proton.

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbours; like the molecules of a liquid which ideally are free to move about but always maintain a fixed inter molecular distance from their nearest molecules.

Assumptions:— The main assumptions of liquid drop model are

(i) The nucleus is supposed to be spherical in shape in its stable state just as a liquid drop is spherical due to the symmetrical forces of surface tension.

(ii) The volume as well as the mass of the nucleus being proportional to the atomic mass number A, the density of nucleus is constant, independent of volume. Similarly the density of a liquid drop is constant, independent of its volume.

(iii) The nucleons are supposed to move about within a spherical enclosure represented by the nuclear potential barrier just as the molecules move about within a spherical drop of the liquid. the shape of which is determined by properties of surface tension,

(iv) The two main property of nuclear forces are their short range and tendency to saturate. This can be deduced from that fact that (a) The binding energy per nucleon is almost a constant quantity (nearly 8 MeV) for all but the lighter nuclei and therefore, the total binding energy of the nucleus is proportional to the total number of nucleons i.e its mass number A and (b) the volume of the nucleus is also proportional to the atomic mass.

v) Just as the latent heat of vaporisation is a constant for a liquid, the binding energy per nucleon is also a constant.

vi) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.

vii) When a small drop of a liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.

Justification:— From the above assumptions we find that the properties of a nucleus are very similar to the properties of a liquid drop. Hence it is justified to call this model of the nucleus as liquid drop model.

Binding energy of a nucleus on the basis of liquid drop model:—

The mass ~~of~~ M of a neutral atom whose nucleus contains Z protons and $N = (A-Z)$ neutrons is given by

$$M(Z, A) = Zm_p + (A-Z)m_n - E_b.$$

$$\text{or } M = Zm_p + Nm_n - E_b.$$

where E_b is the binding energy. The total binding energy of nucleus is contributed by the following factors.

1. Volume energy:— Suppose the energy associated with each nucleon-nucleon bond has a value U. The energy is really because attractive nuclear forces are involved but is usually taken as positive for convenience. Because each bond energy U is shared by two nucleons, each has a binding energy $\frac{1}{2}U$. When an assembly of spheres of the same size is packed together into the smallest volume each interior sphere has 12 other spheres in contact with it, and hence has a binding energy $12 \times \frac{1}{2}U = 6U$. If all the A nucleons in the nucleus are supposed to be in the interior, the total binding energy of the nucleus would be $E_b = 6UA = a_v A$.

where a_v is a constant having a value 14 MeV.
This energy is known as volume energy.

2. Surface energy:— The nucleons on the surface of the drop interact only with nucleons on the side. The above expression which has been derived on the assumption that all nucleons are being equally attracted on all sides, has to be reduced by a factor proportional to the number of nucleons on the surface. The radius of the nucleus is given by

$$R = r_0 A^{1/3}.$$

$$\therefore \text{Surface area} = 4\pi R^2 = 4\pi r_0^2 A^{2/3}$$

$$\therefore \text{Surface energy } E_s = -K 4\pi r_0^2 A^{2/3} = -a_s A^{2/3}$$

where a_s is a constant $= 13$ MeV.
The negative sign indicates that this energy contribution is in a direction opposing to the volume energy, because the nucleons on the surface are not as strongly bound as the nucleons inside. Since the relative number of surface nucleons is pretty high for light nuclei, the surface energy factor is quite large in their case.

3. Coulomb energy:— A nucleus contains Z protons which can form $\frac{Z(Z-1)}{2}$ pairs. The electronic repulsion between each pair of protons in the nucleus contributes towards decreasing of its binding energy. The energy due to ~~repul-~~ repulsion in protons is known as Coulomb energy. Thus the Coulomb energy E_c of a nucleus is the work done to bring together Z protons from infinity into a spherical aggrigate the size of the nucleus. The potential energy for a pair of protons at a distance r apart.

$$V = \frac{e^2}{4\pi\epsilon_0 r}.$$

$$\therefore E_c = \frac{Z(Z-1)}{2} \frac{e^2}{4\pi\epsilon_0 r} = \frac{Z(Z-1)e^2}{8\pi\epsilon_0}\left(\frac{1}{r}\right)_{av}.$$

where $\left(\frac{1}{r}\right)_{av}$ is the value of $\left(\frac{1}{r}\right)$ averaged over all the pair. If the pairs are uniformly distributed through out the nucleus of radius R then $\left(\frac{1}{r}\right)_{av} \propto \frac{1}{R}$ and hence proportional to $\frac{1}{A^{1/3}}$.

$\therefore$ Coulomb energy $E_c \propto \frac{Z(Z-1)e^2}{8\pi\epsilon_0} \frac{1}{A^{1/3}}$

$$= -a_c \frac{Z(Z-1)}{A^{1/3}}$$

Where a_c is an unknown constant $= 0.60$ MeV. The Coulomb energy is negative because it arises from an effect which opposes nuclear stability.

4. Asymmetry energy:— In heavy nuclei the number of neutron $N > Z$ the number of proton. In light nuclei $N = Z$ and these nuclei are highly stable. As the number of neutrons increases, the nucleus acquires an asymmetrical character due to which a force comes in to play which reduces the volume energy. This reduction in energy is directly proportional to the square of the excess of neutrons over proton and inversely proportional to the total number of nucleons.

$$\therefore E_s \propto \frac{(N-Z)^2}{A} = -a_a \frac{(N-Z)^2}{A} = -a_a \frac{(A-2Z)^2}{A}$$

Where a_a is constant $= K(a_v - \cancel{a_s}\ldots)$ and $K = 17826$.

The value of a_a can be taken as 19 MeV for most of the cases.

5. Pairing and shell energy:— Nucleons have a tendency to exist in pairs. Hence the nuclei with even number Z of protons and even number N of neutrons are highly stable, even Z-odd N or odd Z-even N are less stable. The nucleons in a nucleus exist in the form of shells which may be filled by a certain number given by relevant selection rules. Taking both the effects together the term contributing towards volume energy is given by $E_p = (\pm 0) a_p A^{-3/2}$ where a_p is a constant and has a value -34 MeV for even Z even A nuclei, zero for odd A-odd N nuclei and $+34$ MeV for even A and odd Z nuclei.

Weizacker's semi-empirical formula:—

Combining all the terms which contributes towards the binding energy of the nucleus, we get Weizacker's semi-empirical binding energy formula which can be written as

$$E_b = E_v + E_s + E_c + E_a + E_p$$

$$= a_v A - a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + (\pm 0) a_p A^{-3/4}$$

$\rightarrow$

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The binding energy per nucleon is given by

$$\frac{E_b}{A} = a_v - \frac{a_s}{A^{1/3}} - a_c\frac{Z(Z-1)}{A^{4/3}} - a_a\frac{(A-2Z)^2}{A^2} (\pm 0) a_p \frac{A^{-3/4}}{A}$$

Semi-empirial mass formula :-

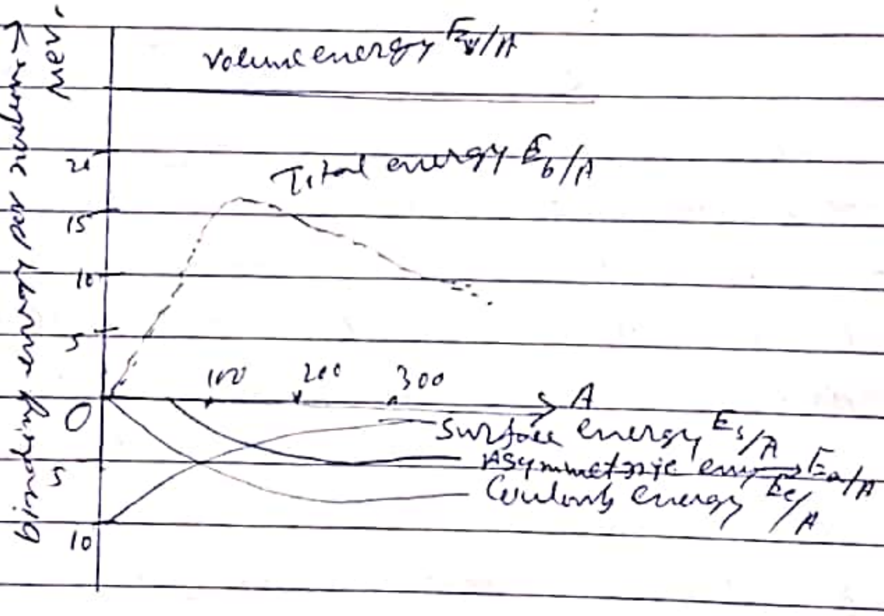
The Complete semiempirial mass formula of the liquid drop model is

$$M(Z,A) = Zm_p + (A-Z)m_n - E_b$$

$$= Zm_p + (A-Z)m_p - a_v A + a_s A^{2/3} + a_c\frac{Z(Z-1)}{A^{1/3}} + a_a\frac{(A-2Z)^2}{A} \cancel{(\pm 0)}$$

$$(\pm 0) a_p A^{-3/4}.$$

Graph :- The Contribution of various energies (except pairing and shell energy) per nucleons towards the binding energy of the nucleus is shown in the fig.



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