Solutions of a Differential Equation:

Definition: Explicit & Implicit solution:

Let us consider the n^{th} order ordinary diff. eqn.

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right) = 0 \quad \longrightarrow (1)$$

Let f be a real valued function defined for all x in a real interval I and having an n^{th} derivative (and hence also all lower order derivatives) for all $x \in I$. The function f is called an explicit solution of the differential equation (1) on I if it fulfills the following two conditions:

(i) $F(x, f(x), f'(x), \ldots, f^n(x))$ is defined $\forall\, x \in I$

and (ii) $F(x, f(x), f'(x), \ldots, f^n(x)) = 0$, $\forall\, x \in I$.

Again, a relation $g(x,y) = 0$ is called explicit sol^n. of (1) if this rel^n. defines at least one real function f of the variable $x \in I$ s.t. this fn. is an explicit sol^n. of (1) on this interval.

Roughly speaking, a solution of the diff. eqn. (1) is a rel^n. – explicit or implicit – bet^n. x and y, not containing derivatives, which identically satisfies the diff. eqn. (1).

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Solution of a Differential Eqn: A relation between x and y, which satisfies a differential eqn., is called a sol^n of the differential eqn.

(i) **General Sol^n:** The relation containing n arbitrary constants which satisfies an ordinary differential eqn. of n^{th} order is called its complete primitive or general solution.

(ii) **Particular solution:** A particular sol^n of a differential eqn. is one obtained from the complete primitive by assigning definite values of the arbitrary constants.

(iii) **Singular sol^n:** – A sol^n, which is not contained in the general sol^n, is called a singular sol^n of the diff. eqn.

Formation of Differential Eqn:

By differentiation and eliminating n arbitrary constants from an equation in x and y, we get a differential eqn. of n^{th} order.

H.W. **Example:** Form a differential from the rel^n

(i) $y = ax + by^2$

(ii) $c(y+c)^2 = x^3$

Note: Number of arbitrary constant is equal to the order of the differential eqn.

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