Differentiable functions
Or
Differentiability of a function

A differentiable function is a continuous function whose derivative exists at all points on its domain. That is, the graph of a differentiable function must have a (non-vertical) tangent line at each point in its domain, be relatively "smooth" (but not necessarily mathematically smooth), and cannot contain any breaks, corners, or cusps. If a function is **differentiable** at a point, then it is also continuous at that point.

Given the function $y = f(x)$, the conditions for continuity of the function at $x = x_0$ are

i) $x = x_0$ must be in the domain of the function f,

ii) y must have a limit as $x \rightarrow x_0$ and

iii) the said limit must be equal to $f(x_0)$. When these are satisfied, we can write

$$\lim_{x \rightarrow x_0} f(x) = f(x_0)$$

(Continuity condition)

When the "limit" concept is applied to different quotient $\frac{\Delta y}{\Delta x}$ as $\Delta x \rightarrow 0$, we deal with the question of whether the function is differentiable at $x = x_0$ i.e., whether the derivative dy/dx exists at $x = x_0$ whether $f'(x_0)$ exists. The term "differentiable" is used here because the process of obtaining the derivative dy/dx is known as differentiation. Since $f'(x_0)$ exists if and only if the limit of $\frac{\Delta y}{\Delta x}$ exists at $x = x_0$ as $\Delta x \rightarrow 0$, the symbolic expression of the differentiability of f is

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

(Differentiability condition)

The two properties- continuity and differentiability are very intimately related to each other, whereas the continuity is a necessary condition for its differentiability. That is, to be differentiable at $x = x_0$, the function must test of being continuous at $x = x_0$. In general, if a function is differentiable at every point in its domain, we may conclude that it must be continuous in its domain. Although differentiability implies continuity, the converse is not true. That is, continuity is a necessary, but not a sufficient condition for differentiability. In sum, all differentiable functions are continuous, but not all continuous functions are differentiable. Therefore, we can say that differentiability is more restrictive condition than continuity as it requires something beyond continuity. Continuity at a point only rules out the presence of a gap, whereas differentiability rules out "sharpness" as well. Most of the specific function employed in economics have the property that they are differentiable everywhere.