found with the help of Laplace, Fourier series also.

iii. The values are of approximation for the physical behavior.

The equation for the motions of the objects which are oscillating can be solved with the help of this concept.

1.3 Superposition of simple Harmonic Oscillations

When we superposition initial conditions corresponding to velocities and amplitudes, resultant displacement of two (or more) harmonic displacements will be algebraic sum of individual displacements at all subsequent times. The principle of superposition holds for any number of simple harmonic oscillations. These may be in same or mutually perpendicular directions, i.e., in two dimensions.

Given equation defines SHM:

$$\frac{d^2x}{dt^2} = -{\omega_0}^2 x \qquad \ldots(1)$$

This is the linear homogeneous equation of second order. Such equation has significant property that sum of its two linearly independent solutions is itself a solution.

Let $x_1(t)$ and $x_2(t)$ respectively satisfy equations

$$\frac{d^2x_1}{dt^2} = -{\omega_0}^2 x_1 \qquad \ldots(2)$$

$$\frac{d^2x_2}{dt^2} = -{\omega_0}^2 x_2 \qquad \ldots(3)$$

Then by adding Eq. 2 and Eq. 3, we get

$$d^2(x_1 + x_2)/dt^2 = -{\omega_0}^2(x_1 + x_2) \qquad \ldots(4)$$

According to principle of superposition, sum of two displacements provided by

$$x(t) = x_1(t) + x_2(t) \qquad \ldots(5)$$

This also satisfies Eq.1. In other words, superposition of two displacements satisfies same linear homogenous differential equation that is satisfied individually by x_1 and x_2.

Let us superpose two collinear (along same line) harmonic oscillations of amplitudes

a_1 and a_2 having frequency ω_0 and phase difference of π. Displacements of the oscillations are provided by

$$x_1 = a_1 \cos \omega_0 t \quad ...(6)$$

And $x_2 = a_2 \cos(\omega_0 t + \pi)$

$$= -a_2 \cos \omega_0 t \quad ...(7)$$

According to principle of superposition, resultant displacement is provided by

$$x(t) = x_1(t) + x_2(t)$$

$$= a_1 \cos \omega_0 t - a_2 \cos \omega_0 t$$

$$= (a_1 - a_2) \cos \omega_0 t \quad ...(8)$$

This represents the simple harmonic motion of amplitude $(a_1 - a_2)$. In particular, if two amplitudes are equal, i.e., $a_1 = a_2$, resultant displacement will be zero at all times.

1.4 Superposition of two collinear harmonic oscillations of equal frequencies

Suppose we have two SHMs of equal frequencies but having different amplitudes and phase constants acting on a system in the x-direction. The displacements x_1 and x_2 of the two harmonic motions, of the same angular frequency ω, differing by phase δ are given by –

$$x_1 = A_1 \sin \omega t$$

$$x_1 = A_1 \sin(\omega t + \delta)$$

There are two methods that can be used to obtain an expression for the resultant displacement due to superposition of the above two harmonic oscillations.

a) Analytical Method

We use the superposition principle which states that the resultant displacement is equal to the vector sum (algebraic sum – because the direction of the two individual oscillations is in the x-direction) of the individual displacements. Therefore, we can write –

$$x = x_1 + x_2$$

$$= A_1 \sin \omega t + A_2 \sin(\omega t + \delta)$$

$$= A_1 \sin \omega t + A_2 \sin \omega t \cos \delta + A_2 \cos \omega t \sin \delta$$

$$= (A_1 + A_2 \cos \delta) \sin \omega t + (A_2 \sin \delta) \cos \omega t$$

$$= C \sin \omega t + D \cos \omega t$$

$$= \sqrt{C^2 + D^2}\left[\frac{C}{\sqrt{C^2 + D^2}} \sin \omega t + \frac{D}{\sqrt{C^2 + D^2}} \cos \omega t\right]$$

where,

$$C = A_1 + A_2 \cos \delta \text{ and } D = A_2 \sin \delta$$

Now we know that the magnitude of $\frac{C}{\sqrt{C^2 + D^2}}$ and $\frac{D}{\sqrt{C^2 + D^2}}$ is less than 1.

Thus, for an angle ε between 0 to 2π, we can have

$$\sin \varepsilon = \frac{D}{\sqrt{C^2 + D^2}} \text{ and } \cos \varepsilon = \frac{C}{\sqrt{C^2 + D^2}}$$

Therefore, we get

$$x = \sqrt{C^2 + D^2}[\cos \varepsilon \sin \omega t + \sin \varepsilon \cos \omega t]$$

or,

$$x = A \sin(\omega t + \varepsilon) \qquad \text{...(1)}$$

where

$$A = \sqrt{C^2 + D^2}$$

$$= \sqrt{(A_1 + A_2 \cos \delta)^2 + (A_2 \sin \delta)^2}$$

$$\therefore \quad A = \sqrt{A_1^2 + 2A_1A_2 \cos \delta + A_2^2} \qquad \text{...(2)}$$

And

$$\tan \varepsilon = \frac{D}{C} = \frac{A_2 \sin \delta}{A_1 + A_2 \cos \delta} \qquad \text{...(3)}$$

Equation (1) shows that the resultant of two collinear simple harmonic motions having the same frequency is itself a simple harmonic motion. The amplitude and phase of the resultant SHM depends on the two individual SHM as well as the phase difference between them.

b) Vector Method

We can arrive at the same results for the superposition of two collinear harmonic oscillations by using vector method too. We will make use of this representation to obtain the resultant of the superposition of two harmonic oscillations here. Let us represent the first SHM by a vector of magnitude A_1 and the second SHM by the vector of magnitude A_2. These two vectors is zero and for the second vector it is equal to the phase difference of δ.

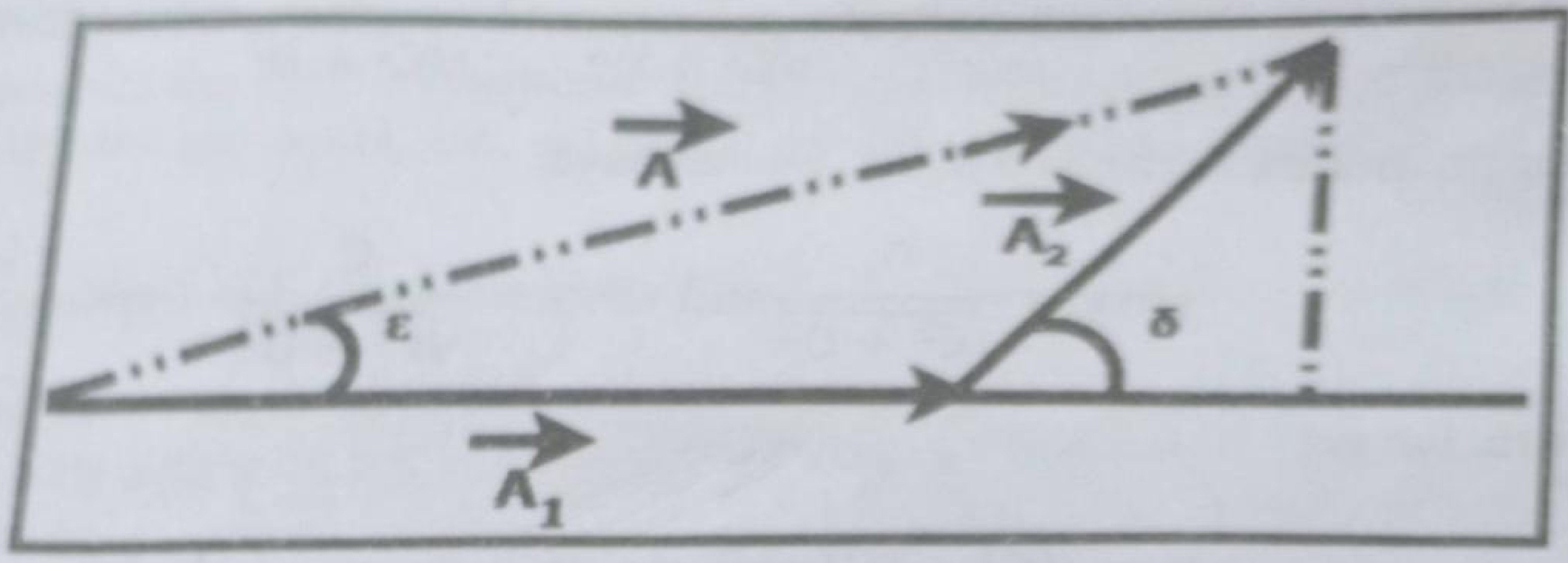

Figure 1.1: Two vector A_1 and A_2 representing SHMs along with the resultant vector A.

Vector $\vec{A}$. This vector represents the resultant SHM. Further, the magnitude of the resultant vector A is given by

$$A = \sqrt{A_1^2 + 2A_1A_2\cos\delta + A_2^2}$$

Which is the same as equation (2)

The resultant $\vec{A}$ makes an angle ε with the x-axis and can be expressed as

$$\tan\varepsilon = \frac{\text{Height}}{\text{Base}} = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

which is the same as equation (3)

The advantage with the vector method is that it can easily be extended to more than two vectors. For example, if we have 3 vectors,

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \delta_1)$$

$$x_3 = A_3 \sin(\omega t + \delta_2)$$

They can be represented as shown in the figure below

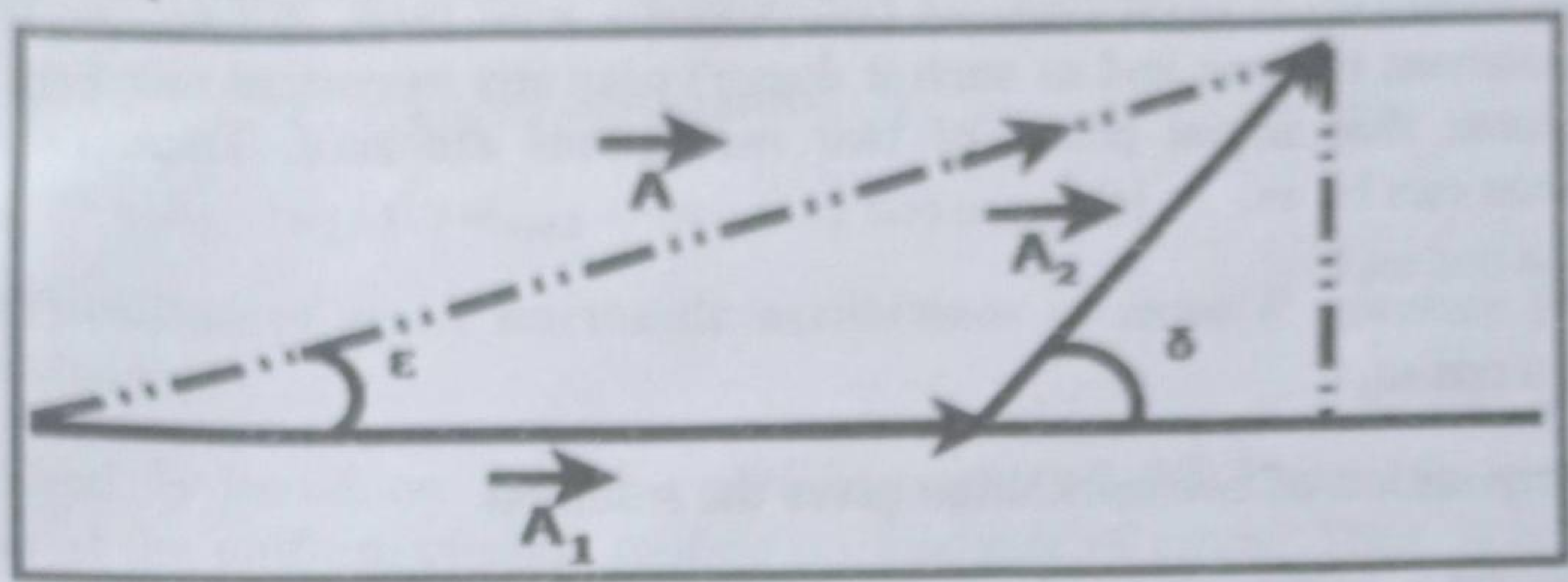

Figure 1.2: There vector representing SHMs along with the resultant vector

The resultant vector is given by

$$x = A\sin(\omega t + \varepsilon)$$

Using trigonometry, we can see that

$$A = \sqrt{(A_1 + A_2\cos\delta_1 + A_3\cos\delta_2)^2 + (A_2\sin\delta_1 + A_3\sin\delta_2)^2}$$

And

$$\tan\varepsilon = \frac{Height}{Base} = \frac{A_2\sin\delta_1 + A_3\sin\delta_2}{A_1 + A_2\cos\delta_1 + A_3\cos\delta_2}$$

1.5 Superposition of two collinear harmonic oscillations of different frequencies

In numerous cases, we have to deal with superposition of two or more harmonic oscillations having different angular frequencies. The microphone diaphragm and human eardrums are simultaneously subjected to different vibrations. For ease, we