Homogeneous Function

A function is said to be a homogeneous function, if multiplication of each of its independent variable by a constant k will alter the value of the function by k times with different degree. Considering the function with two independent variables

$Z = f(x, y)$

is said to be homogeneous of degree r if multiplication of each of its independent variable by a constant k will alter the value of the function by the proportion k^r. In general, k can take any value and in economic application the constant k usually taken to be positive value as most economic variables do not admit negative values.

The power of the constant k in the transformed function defines the degree of homogeneity of the function.

If in a particular case $r = 0$, then the function is said to be homogeneous of degree zero. In this case, the value of the function will not affect at all by the equal proportionate change in all the independent variables.

If $r = 1$, then the function is homogeneous of degree one i.e., multiplication of each variable by k will alter the function exactly k fold as well. This is also known as linear homogeneous function.

If $r = 2$, then the function is said to be homogeneous of degree two i.e., homogeneous of degree greater than one. It is also known as non linear homogeneous function.

Economical Examples

Constant return to scale - production function which is homogenous of degree $r = 1$.

Increasing return to scale - production function which is homogenous of degree $r > 1$.

Decreasing return to scale - production function which is homogenous of degree $r < 1$.

Homothetic Function

In mathematics, a homothetic function is a monotonic transformation of a function which is homogeneous. That is, f is a homothetic function provided that for all (x, y) in a domain D,

$[\, f(x) = f(y), t > 0]$ implies $f(tx) = f(ty)$

A homogeneous function f of any degree *k* is homothetic. But not all homothetic functions are homogeneous. It must be mention that homothetiticy is an ordinal property.